

# MOLDMATE VERIFICATION TOOLKIT

## USER'S GUIDE for `chainLineMarker` and `watermarkMarker`

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## 1 Summary

This document introduces the current version<sup>1</sup> of two software tools - `chainLineMarker` and `watermarkMarker` - that facilitate verification of moldmate status of antique laid paper from images that reveal their inner structural properties - chain lines and watermarks - with sufficient clarity and resolution.

`chainLineMarker` uses mouse-driven placement of a horizontal green stripe across vertical chain lines and allows the user to mark its intersections with the chain lines. The collected chainspace sequence is a prime indicator of moldmate status. A pair of chainline patterns from two separate sheets of antique laid paper must have a close match for the pair to have been made on the same mold and therefore qualify as moldmates.

`watermarkMarker` uses mouse-driven placement of marks at several pairings of key points on a watermark creating a group of line segments that calibrate the shape of the watermark. A well-chosen set of such line segments can provide a unique characterization of a watermark that must be closely matched to signify identical watermarks possessed by moldmates.

The remainder of this document provides a tutorial introduction to the (present) capabilities of these tools in the examination of radiographs of Rembrandt's prints as a sequence of operations each followed by observation of their results. Creation of these tools and this user's guide is being funded by a grant from the Getty Foundation (ORG-201943572). These tools were designed so the technical expertise needed to successfully utilize them does not exceed that required to utilize Photoshop. One bit of advice: The larger the screen used the easier these tools are to employ.

## 2 Strategy for Multi-step Moldmate Verification

The two tools described in this user's guide are part of an evolving moldmate verification strategy that has six steps, each with its own specialized technical approach and tools.

- **STEP ONE** :: Obtain digital images (transmitted light, radiographs) of sufficient resolution, clarity, and contrast.
  - \* Radiographs revealing the inner structural properties of antique laid papers often suffer from a constricted dynamic range and require some processing to enhance their contrast. Contrast and brightness adjustment capabilities are provided in `chainLineMarker` and `watermarkMarker`.

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<sup>1</sup>as Computable Document Format (.cdf) files programmed in Mathematica by William A. Sethares for use with Mathematica's free Wolfram Player (version 12), which can be downloaded from <https://www.wolfram.com/player/>.

- \* The inner structural properties appearing in transmitted light images are often obscured by the opaque surface images. A novel algorithm for surface image removal from transmitted light images (which we have called de-noising) was developed for our study of Leonardo's papers and reported in William A. Sethares, Margaret Holben Ellis, and C. Richard Johnson, Jr., "Computational Watermark Enhancement in Leonardo's *Codex Leicester*," *Journal of American Institute of Conservation*, published online March 3, 2020, <https://sethares.engr.wisc.edu/paperspdf/LeonardoJAIC2020.pdf>. Operation of the software achieving this surface image removal currently requires some expertise in digital image processing. We are continuing to develop this software in an attempt to reduce user expertise to the Photoshop level.
- STEP TWO :: Use visible distinguishing features of watermarks to separate initial set into smaller subgroups.
  - \* The decision tree concept was applied in a previous project - the WIRE Project at Cornell funded by the National Endowment for the Humanities and the National Science Foundation - to achieve rapid, confident identification of watermarks in Rembrandt's etchings. This technique is described in C. R. Johnson, Jr., "Decision Trees for Watermark Identification in Rembrandt's Etchings," to appear in the *Journal of Historians of Netherlandish Art*.
- STEP THREE :: Use chainspace pattern matching to further sub-divide subgroups and provide a necessary, but not a sufficient, condition for moldmate verification.
  - \* `chainLineMarker`, developed with support from the Getty Foundation and described in this document, achieves this.
- STEP FOUR :: Use laid line density pattern matching to further identify moldmates.
  - \* A Fourier-transform-based tool for creating laid line density maps created with the support of the Getty Foundation exploits an approach similar to that adopted in our weave density maps developed for canvas rollmate analysis described in C. Richard Johnson, Jr. and William A. Sethares, eds., *Counting Vermeer*, RKD Studies, 2017 (<http://countingvermeer.rkdmonographs.nl/>). We are continuing to develop this software for laid line density map creation in an attempt to reduce user expertise to the Photoshop level.
- STEP FIVE :: Use dimensional features of watermarks to further sub-divide and verify moldmate pairings.
  - \* `watermarkMarker`, developed with support from the Getty Foundation and described in this document, achieves this.
- STEP SIX :: Construct watermark overlay of presumed moldmates.
  - \* Currently, the effort needed to produce a looping video of one watermark and chain line pattern superimposed on another while one fades and then reappears as the other fades requires significant development time and digital image processing expertise. A sample video of a pair of moldmates from separate codices of Leonardo da Vinci can be found at <https://sethares.engr.wisc.edu/threads/crossCodexEagleAnimation.gif>.

### 3 Guided Tour: chainLineMarker

Presuming previous installation of the free Wolfram CDF player, you are to begin with the following files<sup>2</sup> in a folder on your computer desktop:

- chainLineMarker8.cdf
- RVR178B112ivBetadpi400.tif

The following sequence of operations introduces **chainLineMarker**, which will compute the chainspace widths between the mouse-click marked intersections of the chain lines and the user-located marking line.

#### A. OPEN PLAYER

Open free Wolfram CDF Player. Choose “Open” from the file menu and open `chainLineMarker.cdf`.

#### B. ENABLE DYNAMICS

Click “Enable Dynamics” button in upper right. A default image will appear as shown in Figure 1.<sup>3</sup>

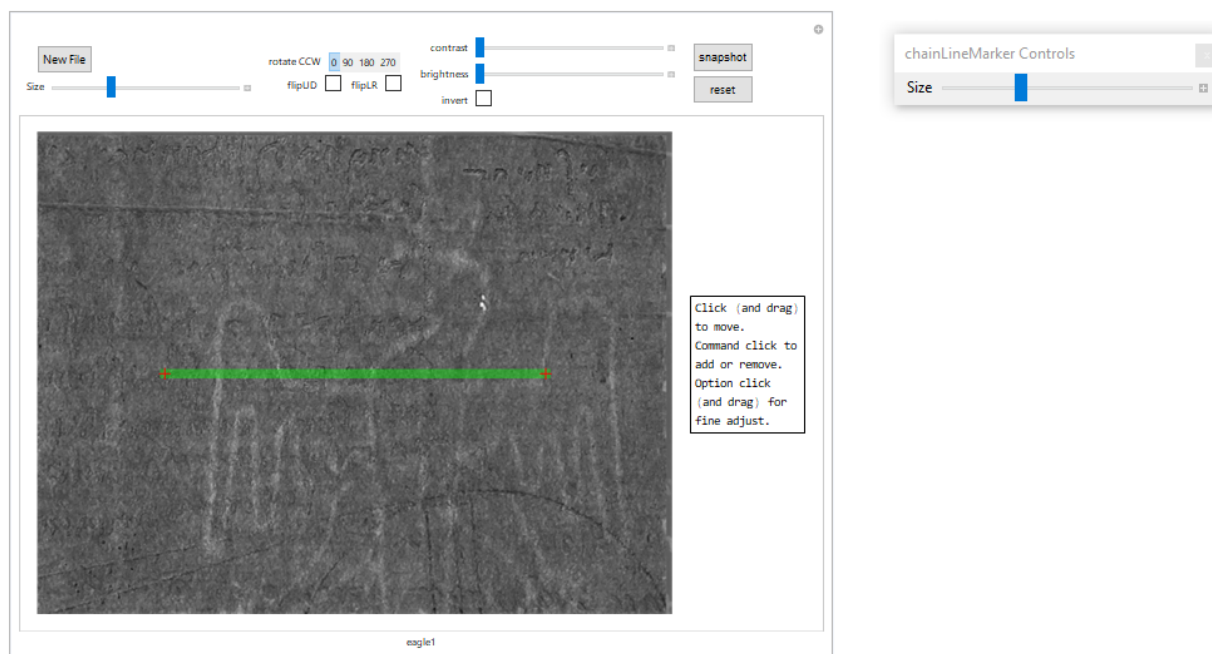


Figure 1: Start Screen for **chainLineMarker**

<sup>2</sup>The authors express their gratitude to the Morgan Library & Museum for permitting the use of their scan of the beta-radiograph RVR178B112ivBetadpi400 of their print by Rembrandt, “Medea, or the Marriage of Jason and Creusa”, B112iv, RVR178B.

<sup>3</sup>The authors are grateful for the permission granted by the Bill and Melinda Gates Collection to use an Eagle watermark from the Leicester codex as the default image. This default image is acquired from processing of a transmitted light image as described in William A. Sethares, Margaret Holben Ellis, and C. Richard Johnson, Jr., “Computational Watermark Enhancement in Leonardo’s *Codex Leicester*,” *Journal of American Institute of Conservation*, published online March 3, 2020, <https://sethares.engr.wisc.edu/paperspdf/LeonardoJAIC2020.pdf>.

### C. OPEN NEW IMAGE FILE

Click “New File” button in upper left. Move to file containing `RVR178B112ivBetadpi400.tif` and double click on tif file. `chainLineMarker` can access only .tif (or .tiff) files.

### D. SIZE IMAGE

To allow the largest viewing area, it is recommended that you go to full screen mode. You can use the “size” slider below the “New File” button to zoom in on the displayed image. The slider is initialized at the location corresponding to a factor of “1”, which displays the image file in its full size. Moving the slider to the right increases the dimensions of the image. Correspondingly, moving it to the left will decrease the dimensions of the image. If you prefer to scale the image dimensions by a specific multiplier, first click the plus at the end of the slider illustrated in the close-up in Figure 2. This will reveal a box with the chosen multiplier. Moving the slider will adjust the multiplier displayed in this box. The multiplier can be set by entering a specific value. Whatever number you put in this box will be a multiplier of the original image’s dimensions. Figure 2 also reveals 6 buttons. From left to right, the first button when clicked reduces the size multiplier by 0.2. The third button increases the size multiplier by 0.2. The second button has play and pause options. Pressing the play option (an arrowhead pointing to the right) moves the slider until the second button is pressed again converting it to the pause symbol (a pair of parallel vertical lines). The direction of the motion is dictated by the sixth button. It has three choices. An arrow pointing the right will result in the slider to move to the right until it reaches the maximum setting (of 3), resets itself to the minimum value (of 0.1) and starts moving to the right again. It will continue until the second button is pressed to pause the slider’s motion. Clicking the 6th button can reveal an arrow pointing left, which when the second button is set to play causes the slider to move to the left to the minimum setting, jump to the maximum setting, move to the left to the minimum, and repeat until stopped with the second (play/pause) button. The third choice for the sixth button is a line with an arrowhead at each end with the one at the right end pointing to the right and at the left end pointing to the left. This results in a back and forth motion from one end of the slider bar to the other. The fourth and fifth slider control buttons increase (double up arrow) or decrease (double down arrow) the speed at which the slider moves. When the image is sized so it is bigger than the screen, returning



Figure 2: Size Slider Control Buttons

to the fixed-location slider below the New File button may be inconvenient. The `chainLineMarker` Controls palette originally in the upper right corner of Figure 1 offers an alternative. A mouse click and drag maneuver can be used to move the palette anywhere on the screen. When the palette is activated by a mouse click anywhere within the palette a red square enclosing a white “x” appears. (Clicking on the red



square will prompt the question: Save changes to “chainLineMarker Controls”. The proper response is to click Cancel.) The slider within the movable palette is operated in the same way as the fixed-location slider below the New File button. Slider location on the palette is independent of that of the fixed-location slider. The square fades to grey when the palette is deactivated by clicking the mouse anywhere outside the palette. Size control is returned to the fixed-location slider by clicking its blue slider or any of the buttons that are present due to a click of the plus at the right end of the slider. You might want to start with a size setting that allows you to observe the entire image, e.g. to view the watermark orientation. A reasonable choice for chain line marking is a setting that shows portions of 4 vertical chain lines enclosing 3 chain spaces. The horizontal position slider at the bottom of the software window can be used to move across the image.

#### E. ROTATE IMAGE

The string of numbered buttons to the right of the New File button labelled with “rotate CCW” can be used to rotate the image in the counterclockwise (CCW) direction. `chainLineMarker` relies on the chain lines being approximately vertical and the green marking line approximately horizontal. If the image's chain lines are originally horizontal, the 90 degree or 270 degree rotation button can be used to rotate the image so the chain lines become vertical and the green marking line crossing the vertical chain lines can be retained as horizontal. The “90”, “180”, and “270” buttons rotate the original image counterclockwise by 90, 180, and 270 degrees, respectively. The “0” setting returns the image to its original orientation.

#### F. FLIP IMAGE

Further image reorientation possibilities are provided by the options below the bar of rotation choices. Clicking the box to the right of flipUD flips the image up-down. The top of the image is pulled forward and down so that the two-dimensional image is now shown upside down from the back. (Note that this differs from a 180 degree rotation, which retains the side being viewed prior to rotation.) Clicking the box to the right of flipLR similarly flips the image left-right. (Note that flipUD followed by flipLR is the same as a 180 degree rotation.) These reorientation functions can be used to obtain a pre-specified watermark orientation.

#### G. CONTRAST AND BRIGHTNESS ADJUSTMENT AND PIXEL VALUE INVERSION

To improve the contrast or brightness of the image to make discernment of the locations of the crossings of the green marking line by the chain lines more precise, you can use the sliders to the right of the rotation number bar. If you wish to invert the black pixels and white pixels to white and black for more comfortable viewing, just click the box to the right of the invert label situated below the brightness and contrast sliders. If your tif image is in color, you may be surprised by the appearance of the inverse colors.

#### H. SCROLL IMAGE

Once the image is adjusted to your liking, use the vertical scroll bar at the right edge of the image to scroll down until a green horizontal line is in view. Use the horizontal scroll bar at the bottom edge of the image to move from the left end of the green marking line with its red plus (“+”) to the right end with its red “+”. Make certain

that the locations where the vertical chain lines cross the green marking line are clearly visible, as in Figure 3.

#### I. RELOCATE LEFT END OF HORIZONTAL GREEN MARKING LINE

Return to the left end of the green marking line and move the mouse-controlled orange marking “x” so it extends the green line so the “x” is centered on the leftmost chain line. Click the mouse and the left end (actually the nearest) “+” will move to that location. If you decide that the “+” is not exactly where you want it, e.g. not exactly centered on the chain line or you wish to move the green marking line up or down, just re-position the “x” and click again. The “+” will relocate.

#### J. RELOCATE RIGHT END OF HORIZONTAL GREEN MARKING LINE APPROXIMATELY PERPENDICULAR TO VERTICAL CHAIN LINES

Repeat step I for the right end of the green marking line and locate the “+” on the rightmost chain line. The green marking line should be approximately perpendicular to the approximately vertical chain lines. A perfectly horizontal marking line will cross perpendicularly perfectly vertical chain lines so that the measured spacings will be the minimum possible distance. However, the chain lines can be tilted and the green marking line need not be perfectly horizontal. The subsequent departure from exactly measuring the smallest distance between chain lines will be corrected by our use of ratios in forming the chain line code defining the chain line pattern of our image<sup>4</sup>.

#### K. TAKING A SNAPSHOT

The snapshot button can be used at any time to record the current workspace.<sup>5</sup> The snapshot will appear as a pdf file in the format `filenameSnapCL-year-month-day-hour-minute-second.pdf` in the folder containing `chainLineMarker`. Figure 3 shows a snapshot of the image after setting the green marking line along with a box providing further instructions. Invoking the snapshot button will also provide a .jpg file of just the watermark and chain line image (without the extra box) with the same name except for the “CL” replaced by “CLImg” and possibly a different value for the final number recording the seconds at the time of its creation. Another .jpg file will also be created of just the information box (or data box once other marks have been made) with the same name except for the “CL” replaced by “CLMat” and possibly a different value for the final number recording the seconds at the time of its creation. A fourth file will also be created as a .txt file with the data recorded in the CLMat image after marks have been made. This .txt file will present the data in various formats as an aid to transcription of the data into a report.

Example list of files created with snapshot in `chainLineMarker` for original image `eagle1.tif`:

- full workspace with (marked) image and data boxes
  - \* label syntax: `filenameSnapCLyear-month-day-hour-minute-sec.pdf`
  - \* example: `eagle1SnapCL2020-05-02-09-07-37.pdf`
- just (marked) image
  - \* label syntax: `filenameSnapCLIMgyear-month-day-hour-minute-sec.jpg`

<sup>4</sup>A bit of trigonometry can be used to verify this statement.

<sup>5</sup>The exception is the `chainLineMarker` Controls palette, which will not appear in a snapshot.

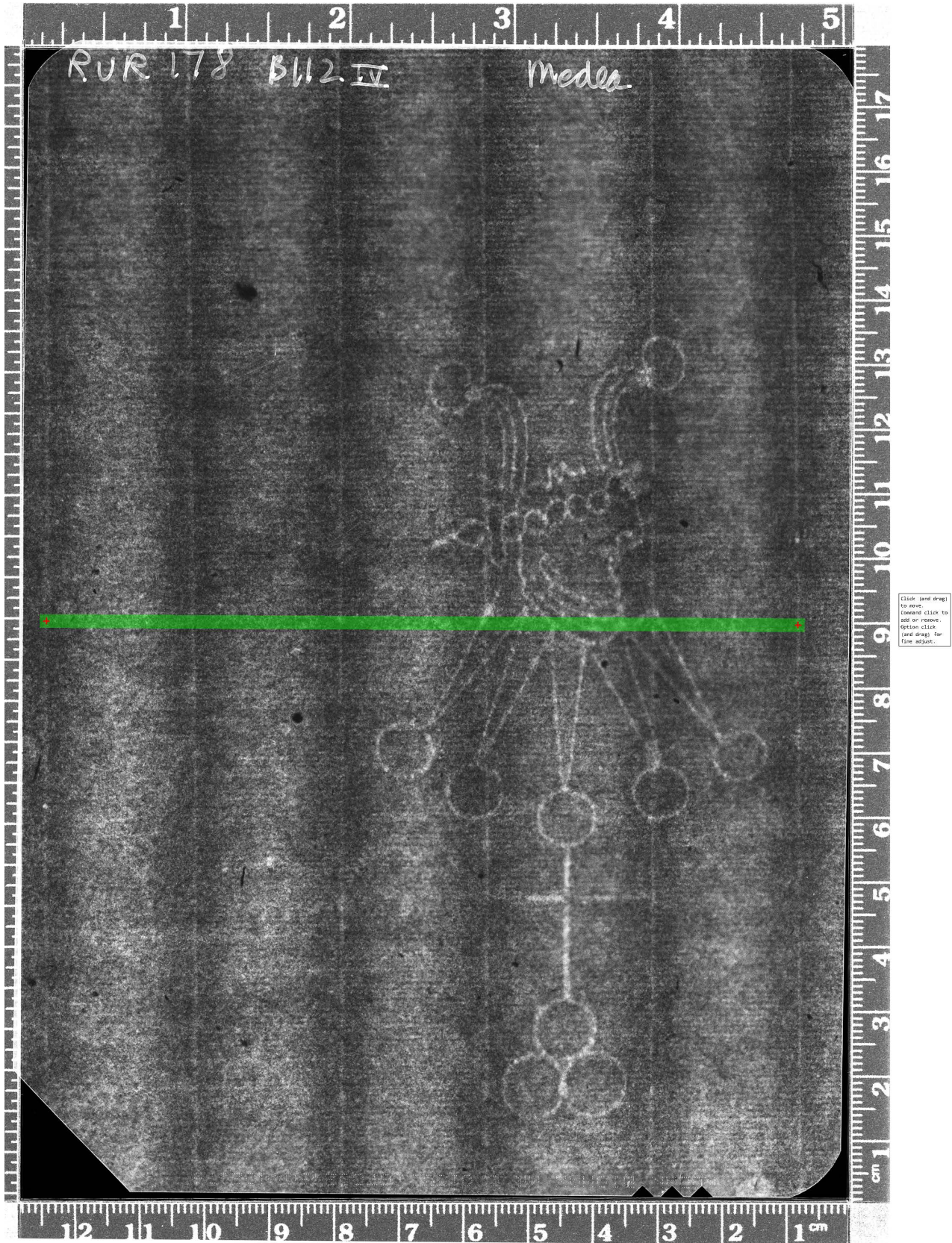


Figure 3: Radiograph with Green Marking Line



- \* example: eagle1SnapCLImg2020-05-02-09-07-39.jpg
- just (data) box(es)
  - \* label syntax: filenameSnapCLMatyear-month-day-hour-minute-sec.jpg
  - \* example: eagle1SnapCLMat2020-05-02-09-07-41.jpg
- data as text in various formats
  - \* label syntax: filenameTextCLyear-month-day-hour-minute-sec.txt
  - \* example: eagle1TextCL2020-05-02-09-07-41.txt

## L. MARKING INTERSECTIONS OF GREEN MARKING LINE AND CHAIN LINES

Moving the mouse anywhere on a chain line within the green marking line and pressing the alt key<sup>6</sup> and clicking the mouse will place a red “+” in the center of the marking line, so all chain line crossing marks are on a straight line. The crossings can be marked in any sequence. Just make certain all are marked. If a mark is not located where desired, just reposition and click the mouse without re-pressing the alt key. If you are having trouble precisely locating the “+”, you can press and hold down the alt key and then press and not release the mouse and drag the “x” until the “+” is located at the desired location. In this mode the “x” will offer finer control as the cursor needs to be moved a substantial distance to relocate the “+” by a small distance.<sup>7</sup> If you want to remove completely a “+”, position the “x” over it, hold the alt key down, and left-click the mouse. If for some reason all but one red “+” (including endpoints) is removed from the green marking line, **chainLine Marker** will automatically reset the green marking line with two “+” endpoints to its starting location.

## M. RESTART, IF DESIRED

A re-start of the marking process at step I can be achieved using the reset button. Pressing the reset button to the right of the contrast and brightness sliders will clear all interior marked points on the green marking line and return the marking line to its starting position. All other settings for size, invert, rotation, flip, brightness, and contrast will be retained.

## N. DIMENSION AND RATIO TABLES

Once more than two chain line crossings have been marked between the end points of the green marking line, two tables will appear in the image. These tables will be updated with each subsequent “+” placed on the marking line. The marked image and the two tables will all appear in a snapshot. Figure 4 records the marked image and the two tables for a marking of all chain line crossings for our example. One table indicates the end points and lengths in pixels of the lines between marked chain line crossings. The lengths of these lines numbered left to right quantify the chainspace widths. The other table lists ratios of the lengths of the correspondingly numbered chainspaces. When a snapshot is taken with a data table (or matrix) of ratios present this data is saved as well in a separate .txt file in various formats to aid transcription. The marked image alone and the data tables alone are also saved as .jpg files, as described above in step K. (The tables are readable in Figure 4 when enlarged, but are small relative

<sup>6</sup>This is for a Windows operating system. For an Apple operating system press the command key.

<sup>7</sup>This is for a Windows operating system. To enable this fine relocation of “x” for an Apple operating system, press and hold down the option key and use the mouse to drag the “x” by pressing and not releasing the mouse until the desired location is reached.

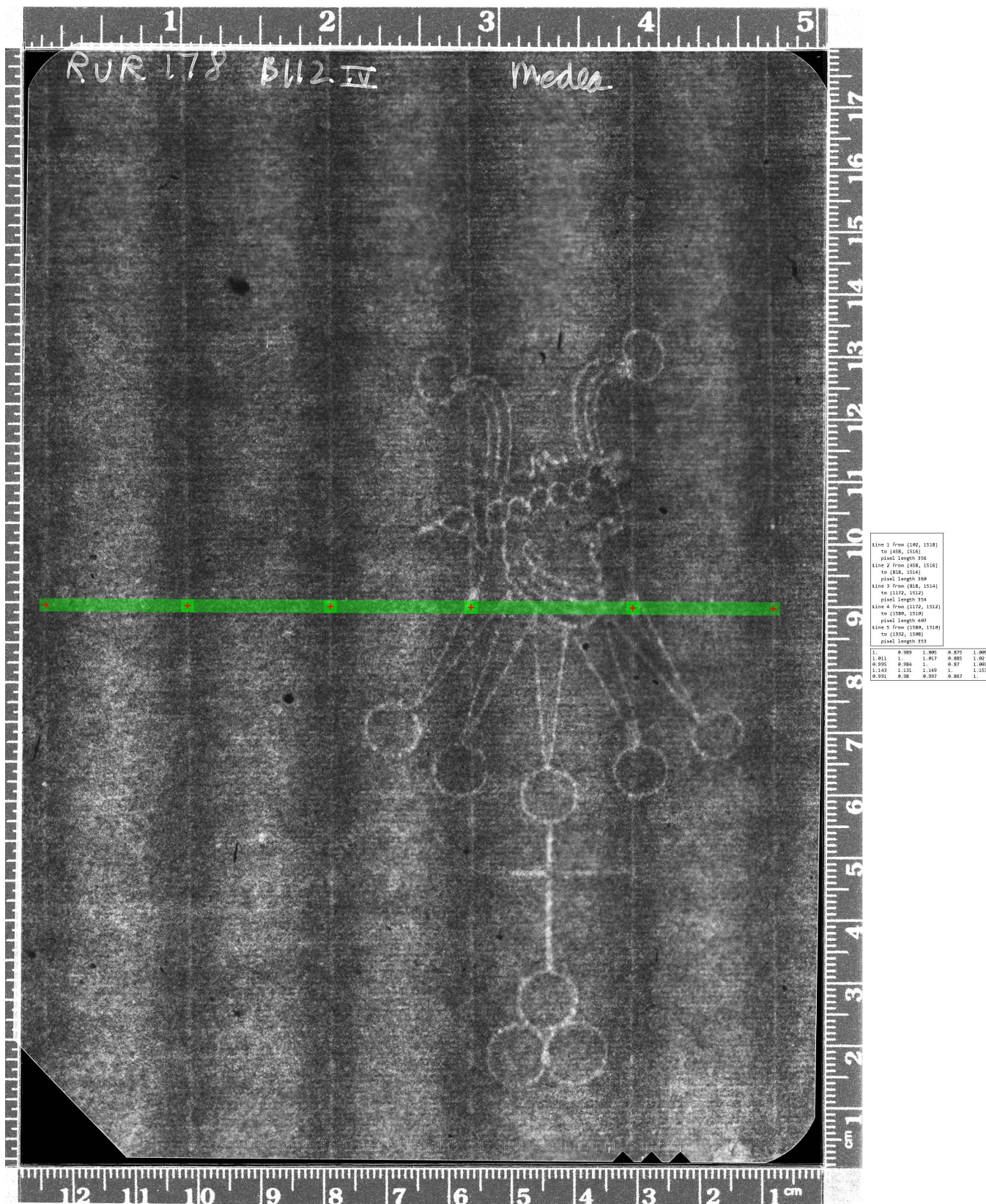


Figure 4: Radiograph with Marked Chain Line Crossings



to the marked radiograph. An enlarged version of the pair of tables from Figure 4 appears in Figure 6.) The top row from left to right is the value of space 1 divided by each of the spaces in sequence, i.e. (space 1)/(space 1), (space 1)/(space2), (space 1)/(space 3), etc. The second row is the width of space 2 divided by each space in sequence, i.e. (space 2)/(space 1), (space 2)/(space2), (space 2)/(space 3), etc. The third row divides space 3 by by each space in the left to right sequence. The specific ratios of chainspaces in each entry in the ratio matrix are illustrated in the following table.

1/1	1/2	1/3	1/4	1/5
2/1	2/2	2/3	2/4	2/5
3/1	3/2	3/3	3/4	3/5
4/1	4/2	4/3	4/4	4/5
5/1	5/2	5/3	5/4	5/5

Thus the “x divided by x” entries on the main diagonal of this matrix from top left to bottom right are all 1.00, as can be seen in Figure 4. The values in the first row second column or 1-2 location, the 2-3 location, 3-4 location, and the 4-5 location are right over left ratios of adjacent spaces, i.e. 1/2, 2/3, 3/4, 4/5. To confirm consider the 4th row - 5th column value of 1.153 in the ratios table. From the line location and pixel length table, line 4 has length 407 and line 5 has length 353. Their ratio of line 4 length over line 5 length is 407/353, which equals 1.153 when rounded to three significant digits. This matches the 4-5 entry of 1.153 in the ratios table. One advantage of ratios is that they remain unchanged if the original analog radiograph is scanned at a different resolution. A second advantage of ratios is that they remain the same even when the spacings are measured with a green marking line not precisely perpendicular to the chain lines<sup>8</sup>.

#### O. CHAIN LINE CODE

The sequence of ratios of chainspaces 1/2, 2/3, 3/4, and 4/5 from the upper diagonal of the ratio matrix, i.e. the 1-2, 2-3, 3-4, 4-5 values highlighted in red in the following table, provide a useful *code* for comparing possible moldmates.

1/1	1/2	1/3	1/4	1/5
2/1	2/2	2/3	2/4	2/5
3/1	3/2	3/3	3/4	3/5
4/1	4/2	4/3	4/4	4/5
5/1	5/2	5/3	5/4	5/5

This code from Figure 4 is 0.989, 1.017, 0.870, 1.153. Two sheets of paper with sufficiently dissimilar codes cannot be moldmates. To compare the chainline codes of two different papers, it is necessary to keep the orientations of the papers consistent. But if measurements are taken with different left/right or different flips, the chain line codes can be taken from a different portion of the ratios matrix. If one marked image has its watermark, and therefore its basic orientation, upside down relative to the other marked image, this will not affect using the upper diagonal of the ratio matrix as providing the comparative code. However, if one image is left-right reversed relative

<sup>8</sup>A bit of trigonometry can be used to verify this statement.



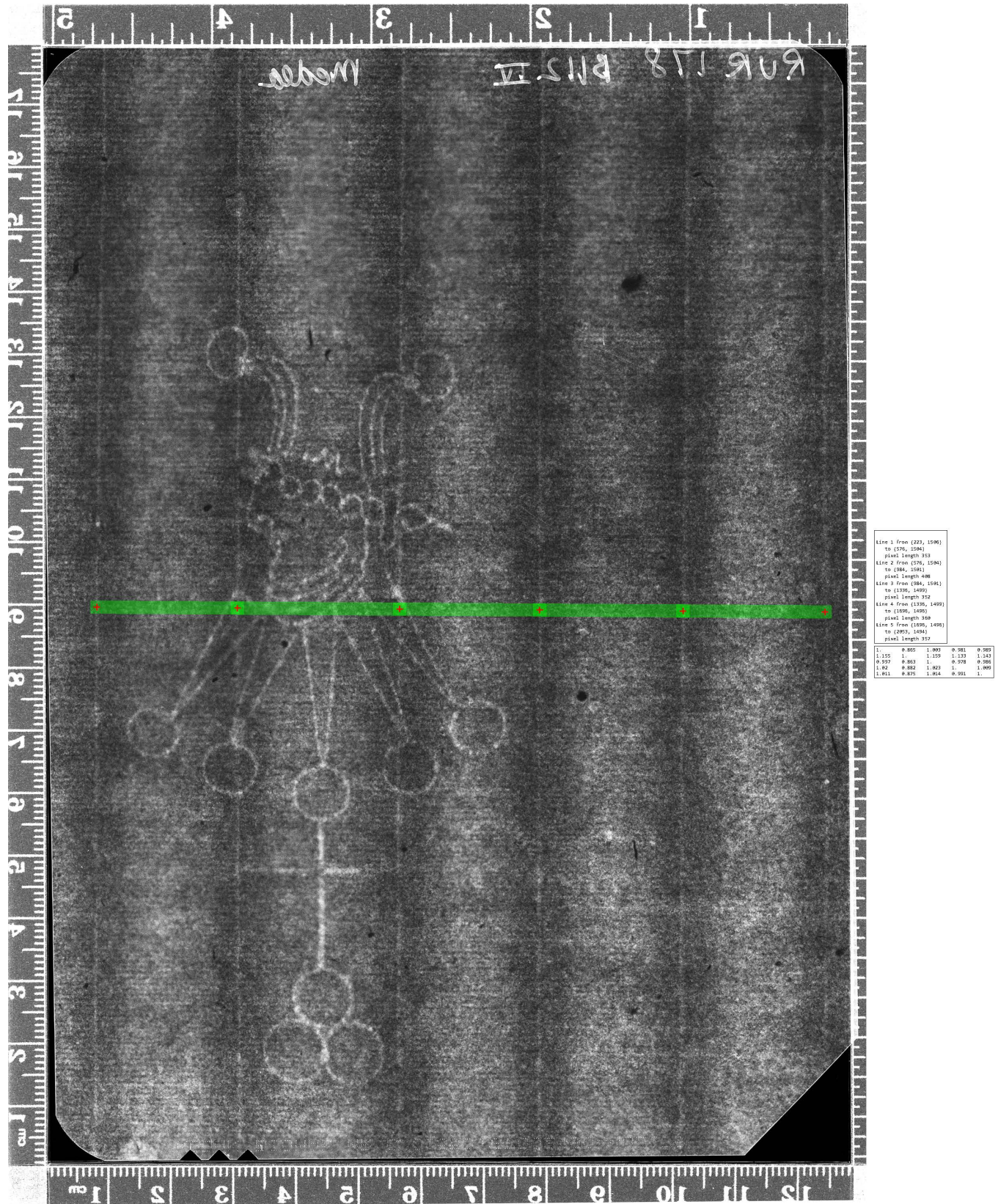


Figure 5: Left-Right Flipped Radiograph with Marked Chain Line Crossings

to the other one of the codes should be taken from the lower diagonal from bottom right to upper left, i.e. matrix locations 5-4, 4-3, 3-2, 2-1, as these provide the ratios 5/4, 4/3, 3/2, and 2/1, as highlighted in green in the following table.

1/1	1/2	1/3	1/4	1/5
2/1	2/2	2/3	2/4	2/5
3/1	3/2	3/3	3/4	3/5
4/1	4/2	4/3	4/4	4/5
5/1	5/2	5/3	5/4	5/5

But if the image had been reversed left to right 5 would become 1, 4 would be 2, 3 would still be 3, 2 would be 4, and 1 would be 5. Then the 5/4, 4/3, 3/2, and 2/1 sequence would become the ratios 1/2, 2/3, 3/4, and 4/5 as desired. You can test this by using the flipLR button to reverse the original image and repeat the marking of the chain line crossings. The result is shown in Figure 5. The ratio tables are expanded for direct comparison in Figure 6. The corresponding numbers will not be exactly the same due

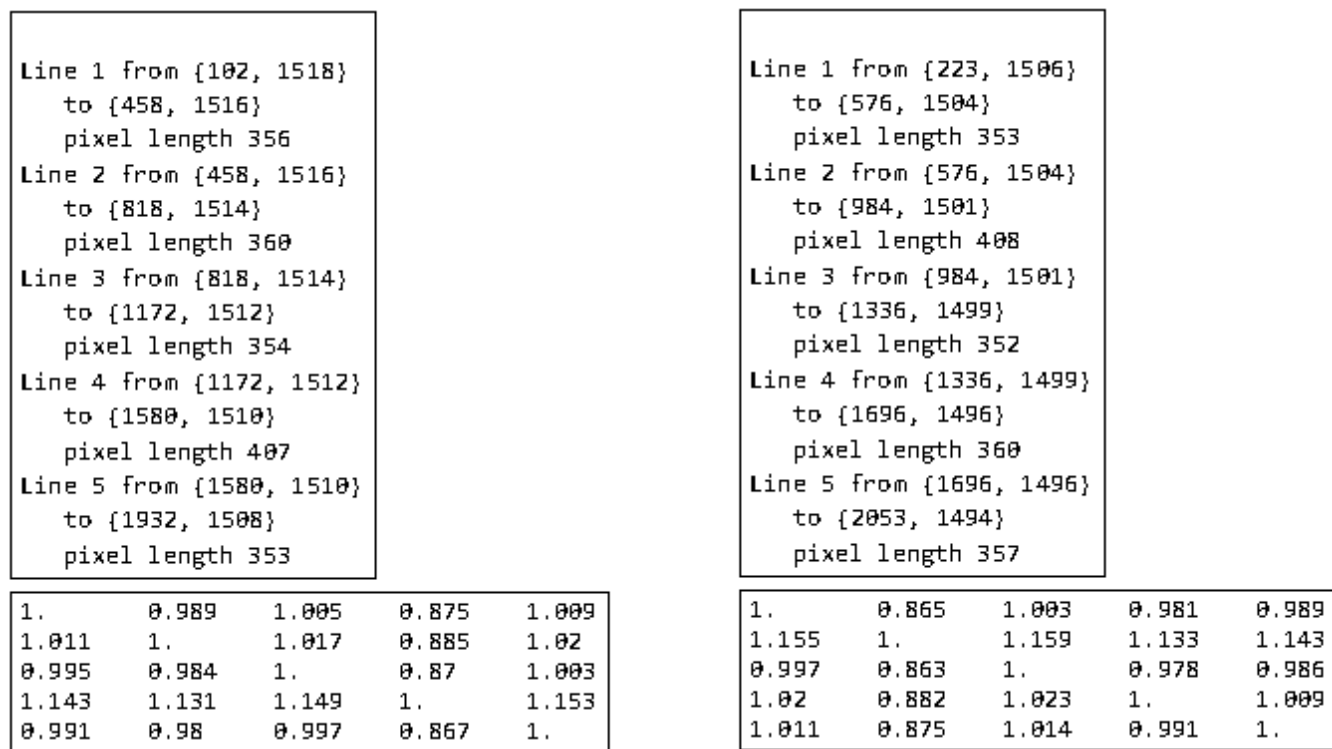


Figure 6: Distance and Ratio Tables from Figures 4 and 5

to marking “errors”. However, note how similar the corresponding chainspace distance measures are. The pixel lengths of the Lines 1 to 5 from Figure 4 recorded in the top table on the left in Figure 6 are 356, 360, 354, 407, and 353. The corresponding pixel lengths of Lines 5 to 1 from Figure 5 recorded in the top table on the right in Figure 6 are 357, 360, 352, 408, and 353. The marking “error” is never more than two pixels. This code from Figure 4 drawn from the upper diagonal entries from upper left to bottom right in the bottom table on the left in Figure 6 when rounded to two decimal places is 0.99, 1.02, 0.87, 1.15. For Figure 5 the corresponding code drawn from the lower diagonal entries from bottom right to upper left in the bottom table on the right in Figure 6 when rounded to two decimal places is 0.99, 1.02, 0.86, 1.16.



The corresponding code values differ by at most 0.01, which provides an estimate of code “error” due to marking “error” that is to be expected with this tool.<sup>9</sup> A text file with the same snapshot name and number is provided of the data in a ratio table each time a snapshot is taken. This information is provided in various formats for easier transcription.

#### P. CLOSING

When your use of `chainLineMarker` is complete and you wish to close it, select “Close” from the pull down “File” menu of the Wolfram Player. In response to the question asking if you want to “Save changes to `chainLineMarker`” click “Don’t save”.

## 4 Guided Tour: watermarkMarker

Presuming previous installation of the free Wolfram CDF player, you are to begin with the following files<sup>10</sup> in a folder on your computer desktop:

- `watermarkMarker10.cdf`
- `RVR178B112ivBetadpi400.tif`

The following sequence of operations introduces `watermarkMarker`, which will allow you to designate a number of line segments between precisely selectable mouse-clicked points in the image of the watermark. The software will provide a matrix of ratios of all pairings of the numbered line segments and for specially marked connected line segments the angle they form will also be computed and provided.

#### A. OPEN PLAYER

Open free Wolfram CDF Player. Choose “Open” from the file menu and open `watermarkMarker.cdf`.

#### B. ENABLE DYNAMICS

Click “Enable Dynamics” button in upper right. A default image will appear as shown in Figure 7.<sup>11</sup>

#### C. OPEN NEW IMAGE FILE

Click “New File” button in upper left. Move to file containing `RVR178B112ivBetadpi400.tif` and double click on it. `watermarkMarker` can access only `.tif` (or `.tiff`) files.

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<sup>9</sup>The maximum code “error” is the same, i.e. never greater than 0.01, if the code from Figure 5 were taken from the upper diagonal of its ratio table as 0.87, 1.16, 0.98, 1.01 when rounded to two digits and the corresponding code for Figure 4 were taken from its lower diagonal as 0.87, 1.15, 0.98, 1.01.

<sup>10</sup>The authors express their gratitude to the Morgan Library & Museum for permitting the use of their scan of the beta-radiograph RVR178B112ivBetadpi400 of their print by Rembrandt, “Medea, or the Marriage of Jason and Creusa”, B112iv, RVR178B.

<sup>11</sup>The authors are grateful for the permission granted by the Bill and Melinda Gates Collection to use an Eagle watermark from the Leicester codex as the default image. This default image is acquired from processing of a transmitted light image as described in William A. Sethares, Margaret Holben Ellis, and C. Richard Johnson, Jr., “Computational Watermark Enhancement in Leonardo’s *Codex Leicester*,” *Journal of American Institute of Conservation*, published online March 3, 2020, <https://sethares.engr.wisc.edu/paperspdf/LeonardoJAIC2020.pdf>.

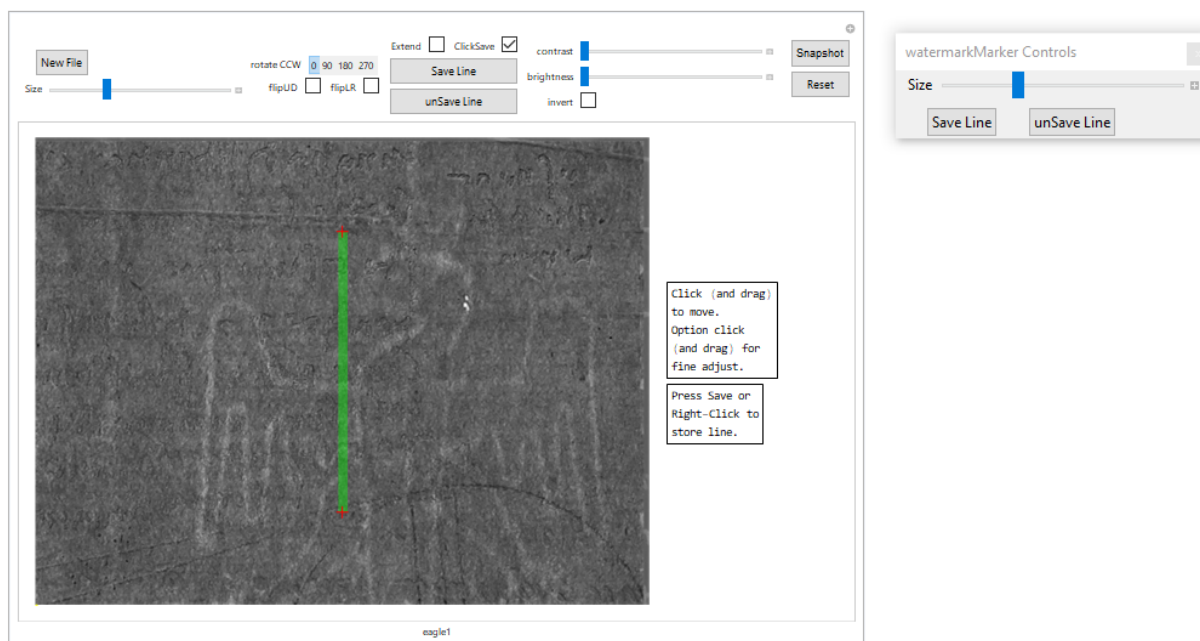


Figure 7: Start Screen for watermarkMarker

#### D. SIZE IMAGE

To allow the largest viewing area, it is recommended that you go to full screen mode. You can use the “size” slider below the “New File” button to zoom in on the displayed image. The slider is initialized at the location corresponding to a factor of “1”, which displays the image file in its full size. Moving the slider to the right increases the dimensions of the image. Correspondingly, moving it to the left will decrease the dimensions of the image. If you prefer to scale the image dimensions by a specific multiplier, first click the plus at the end of the slider illustrated in the close-up in Figure 2. This will reveal a box with the chosen multiplier. Moving the slider will adjust the multiplier displayed in this box. The multiplier can be set by entering a specific value. Whatever number you put in this box will be a multiplier of the original image’s dimensions. Figure 2 also reveals 6 buttons. From left to right, the first button when clicked reduces the size multiplier by 0.2. The third button increases the size multiplier by 0.2. The second button has play and pause options. Pressing the play option (an arrowhead pointing to the right) moves the slider until the second button is pressed again converting it to the pause symbol (a pair of parallel vertical lines). The direction of the motion is dictated by the sixth button. It has three choices. An arrow pointing the right will result in the slider to move to the right until it reaches the maximum setting (of 3), resets itself to the minimum value (of 0.1) and starts moving to the right again. It will continue until the second button is pressed to pause the slider’s motion. Clicking the 6th button can reveal an arrow pointing left, which when the second button is set to play causes the slider to move to the left to the minimum setting, jump to the maximum setting, move to the left to the minimum, and repeat until stopped with the second (play/pause) button. The third choice for the sixth button is a line with an arrowhead at each end with the one at the right end pointing to the right and at the left end pointing to the left. This results in a back and forth motion from one end of the slider bar to the other. The fourth and fifth slider control buttons increase (double up arrow) or decrease (double down arrow) the speed

at which the slider moves. When the image is sized so it is bigger than the screen, returning to the fixed-location slider below the New File button may be inconvenient. The watermarkMarker Controls palette originally in the upper right corner of Figure 7 offers an alternative. A mouse click and drag maneuver can be used to move the palette anywhere on the screen. When the palette is activated by a mouse click anywhere within the palette a red square enclosing a white “x” appears. (Clicking on the red square will prompt the question: Save changes to “watermarkMarker Controls”. The proper response is to click Cancel.) The slider within the movable palette is operated in the same way as the fixed-location slider below the New File button. Slider location on the palette is independent of that of the fixed-location slider. The square fades to grey when the palette is deactivated by clicking the mouse anywhere outside the palette. Size control is returned to the fixed-location slider by clicking its blue slider or any of the buttons that are present due to a click of the plus at the right end of the slider. A reasonable choice for watermark marking is a setting that allows viewing of the entire watermark within the screen. Typically for large tif files a setting less than one is needed to achieve this.

#### E. ROTATE IMAGE

The string of numbered buttons to the right of the New File button labelled with “rotate CCW” can be used to rotate the image in the counterclockwise (CCW) direction. The “90”, “180”, and “270” buttons rotate the original image counterclockwise by 90, 180, and 270 degrees, respectively. The “0” setting returns the image to its original orientation.

#### F. FLIP IMAGE

Further image reorientation possibilities are provided by the options below the bar of rotation choices. Clicking the box to the right of flipUD flips the image up-down. The top of the image is pulled forward and down so that the two-dimensional image is now shown upside down from the back. (Note that this differs from a 180 degree rotation, which retains the side being viewed prior to rotation.) Clicking the box to the right of flipLR similarly flips the image left-right. (Note that flipUD followed by flipLR is the same as a 180 degree rotation.) These reorientation functions can be used to obtain a pre-specified watermark orientation.

#### G. CONTRAST AND BRIGHTNESS ADJUSTMENT AND PIXEL VALUE INVERSION

To improve the contrast or brightness of the image to make discernment of the watermark more precise, you can use the sliders to the right of the column of Extend line, Save Line, and UnSave Line operators. If you wish to invert the black pixels and white pixels to white and black for more comfortable viewing, just click the box to the right of the invert label situated below the brightness and contrast sliders. If your tif image is in color, you may be surprised by the appearance of the inverse colors.

#### H. TAKING A SNAPSHOT

To save a copy of the current workspace at any time press the Snapshot button to the right of the brightness and contrast sliders.<sup>12</sup> The snapshot will appear as a pdf file in the format filenameSnapWM-year-month-day-hour-minute-second.pdf in the folder containing watermarkMarker. Figure 8 shows the snapshot of the starting location of

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<sup>12</sup>The exception is the watermarkMarker Controls palette, which will not appear in a snapshot.

the green line along with a box providing further instructions. Invoking the snapshot button will also provide a .jpg file of just the watermark and chain line image (without the extra box) with the same name except for the “WM” replaced by “WMImp” and possibly a different value for the final number recording the seconds at the time of its creation. Another .jpg file will also be created of just the information box (or data box once other marks have been made) with the same name except for the “WM” replaced by “WMMat” and possibly a different value for the final number recording the seconds at the time of its creation. A fourth file will also be created as a .txt file with the data recorded in the WMMat image after marks have been made. This .txt file will present the data in various formats as an aid to transcription of the data into a report.

Example list of files created with snapshot in `watwermarkMarker` for original image `eagle1.tif`:

- full workspace with (marked) image and data boxes
  - \* label syntax: `filenameSnapWMyear-month-day-hour-minute-sec.pdf`
  - \* example: `eagle1SnapWM2020-05-02-09-41-58.pdf`
- just (marked) image
  - \* label syntax: `filenameSnapWMImpyear-month-day-hour-minute-sec.jpg`
  - \* example: `eagle1SnapWMImp2020-05-02-09-41-58.jpg`
- just (data) box(es)
  - \* label syntax: `filenameSnapWMMatyear-month-day-hour-minute-sec.jpg`
  - \* example: `eagle1SnapWMMat2020-05-02-09-41-59.jpg`
- data as text in various formats
  - \* label syntax: `filenameTextWMyear-month-day-hour-minute-sec.txt`
  - \* example: `eagle1TextWM2020-05-02-09-07-41.txt`

## I. MARKING ENDPOINTS AND SAVING A LINE

To begin marking the end points of line segments connecting two precisely locatable points, click the orange “x” at one of the desired end points. The red “+” on the green line closest to the location of the “x” will relocate at the site of the “x”. To shift the relocated “+” to a more precise match of the desired point, or to a different choice for that desired point, simply click the “x” again at the improved spot. If you are having trouble precisely locating the “+”, you can press and hold down the alt key and then press and not release the mouse and drag the “x” until the “+” is located at the desired location. This provides finer control as the “x” will need to be moved a substantial distance to relocate the “+” by a small distance.<sup>13</sup> The other end of the green line can be moved similarly. Remember that the closest “x” at the ends of the green line will be relocated by clicking the mouse at the “x” location. To save a marked line click the Save Line button. If you find it more convenient to right-click your mouse in order to save a line, you should activate the Click Save option to the right of the Extend option by clicking the box to the right of Click Save. Note that this option is activated by default at the start-up of `watermarkMarker`, as shown in Figure 8. Another possibility is offered by the movable `watwermarkMarker` Controls palette. The snapshot displayed

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<sup>13</sup>This is for a Windows operating system. To enable this fine relocation of “x” for an Apple operating system, press and hold down the option key and use the mouse to drag the “x” by pressing and not releasing the mouse until the desired location is reached.





Figure 8: Starting Image for Line Marking

in Figure 9 shows the first marked line. Note the appearance of a table listing the line 1 endpoint locations and length in pixels. When a snapshot is taken with a data table present the data in this table is saved as well in a separate .txt file in various formats to aid transcription. The marked image alone and the data tables alone are also saved as .jpg files, as described above in step H.



Figure 9: Snapshot after Marking First Line

#### J. MARKING CONNECTED LINES FOR ANGLE MEASUREMENT

The option exists for computing the angle between two lines by moving only one end of the first line to a new location. The number 2 line in Figure 10 is obtained by



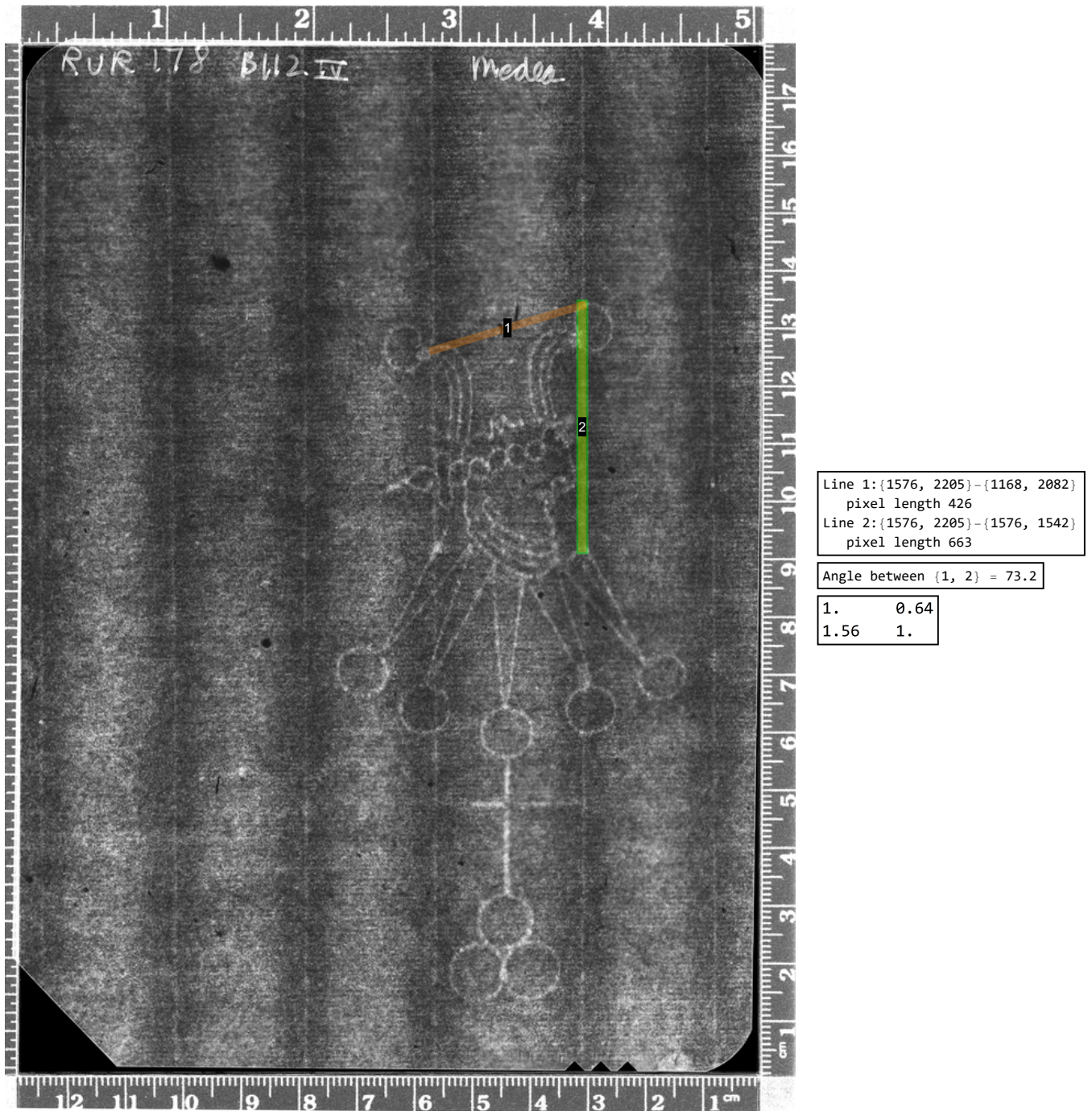


Figure 10: Snapshot after Second Line is Saved

moving the left, lower end point of Line 1. This can be done by putting the “x” near the left end of Line 1, left-clicking the mouse, moving the “x” to a point nearer to the relocated end point and nearer to its final desired location, and left-clicking the mouse again. This relocation (which can also be achieved by click and drag) must be done so that each subsequent clicked location is closer to the end point being moved than to the other end point. When the new line segment is located as desired, click Save Line. The result appears in the snapshot in Figure 10, where the upper end point of Line 2 is the unmoved upper end point of Line 1. Note the appearance of a table of angles and a matrix of length ratios. The values appearing in the tables are also saved in a .txt file. The recorded angle is the number of degrees in the acute (less than 180 degrees) angle formed by the connected pair of line segments. The marked image alone and the data tables alone are also saved as .jpg files, as described above in step H.

#### K. MOVING BOTH END POINTS FOR UNCONNECTED LINES

If both endpoints of the previous marked line are moved, no angle data is recorded. Figure 11 shows this with the creation of Line 6. Only the angles formed by successive connected line segments are listed in the table of angles.

#### L. MARKED LINE EXTENSION

A new green line can be extended across the image by clicking the extend box above the Save Line button. For the new green line shown in Figure 12, clicking the extend box extends the green line marked by two endpoints in both directions with a yellow line to the edges of the image as shown in Figure 13. The endpoints of the green line can then be relocated with mouse clicks to desired locations along the yellow line. This is illustrated in Figure 14 where the left end point of the new green line is extended to its intersection with the chain line. Once this newest line is saved its dimension and ratio data is added to the tables.

#### M. DISCARDING A SAVED LINE

If at some point in the line saving process you wish to discard a previously saved line, you can use the unSave Line button below the Save Line button. Another possibility is offered by the unSave Line button on the movable watermarkMarker Controls palette. The unSave Line button removes lines in reverse order. Assume you have saved 4 lines, but now want to remove line 3. You can press the unSave Line button twice, which removes line 4 and line 3 from the table. Actually the orange version of line 3 is removed from the image. But the green version of line 4 (which doesn't turn orange until saving line 5) is still in place. So, you could re-save it as the new line 3. The tables adjust accordingly. In this scenario, if you want to remove just line 2, after clicking unSave Line 3 times, you will not be able to recover the old line 3.

#### N. Restart, if Desired

If you want to re-start the marking process at step I, use the Reset button below the Snapshot button. Pressing the Reset button will clear all marked lines and return the marking line to its starting position. All other settings for size, invert, rotation, flip, brightness, and contrast will be retained.

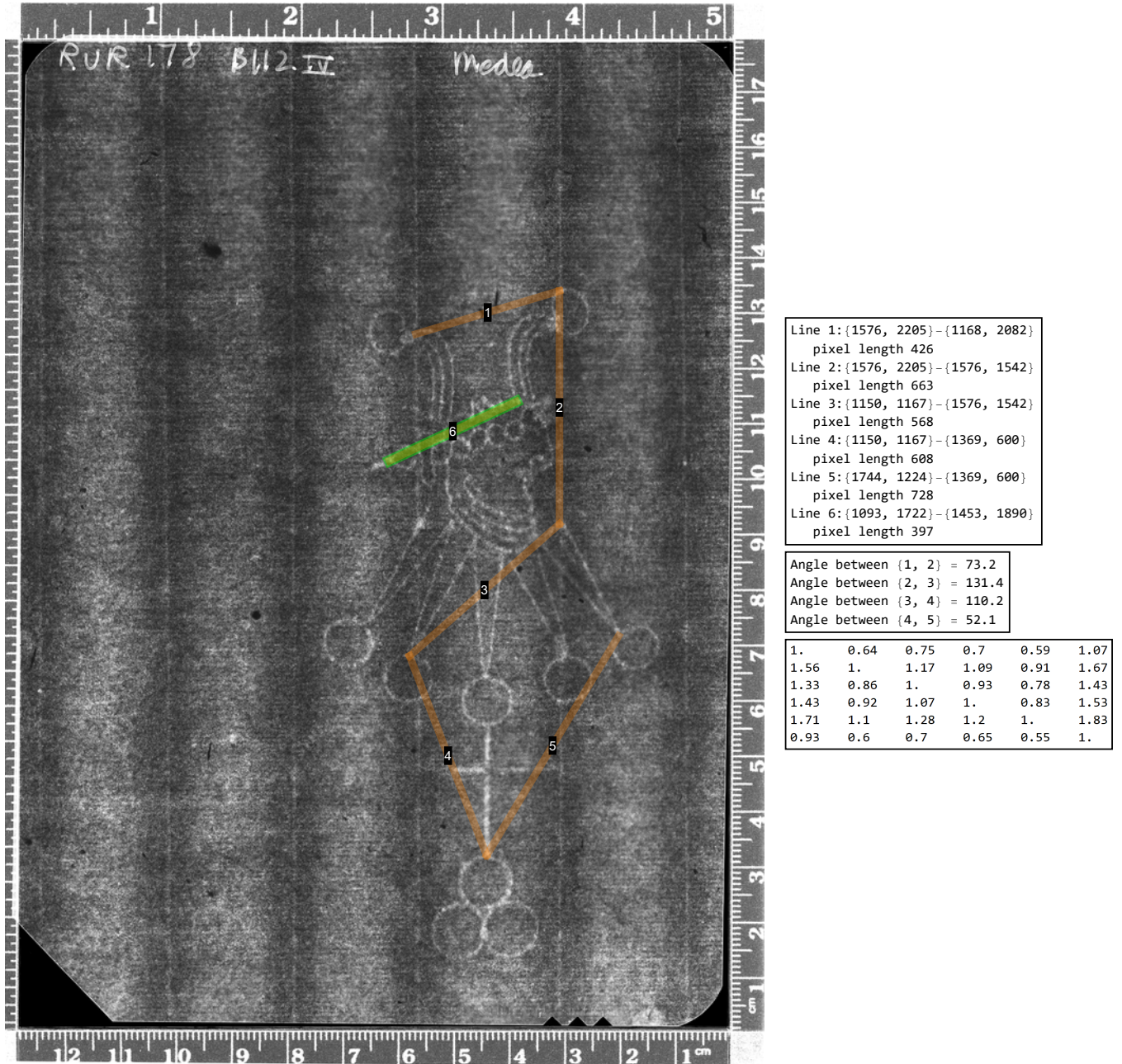


Figure 11: Snapshot after Sixth Line is Saved



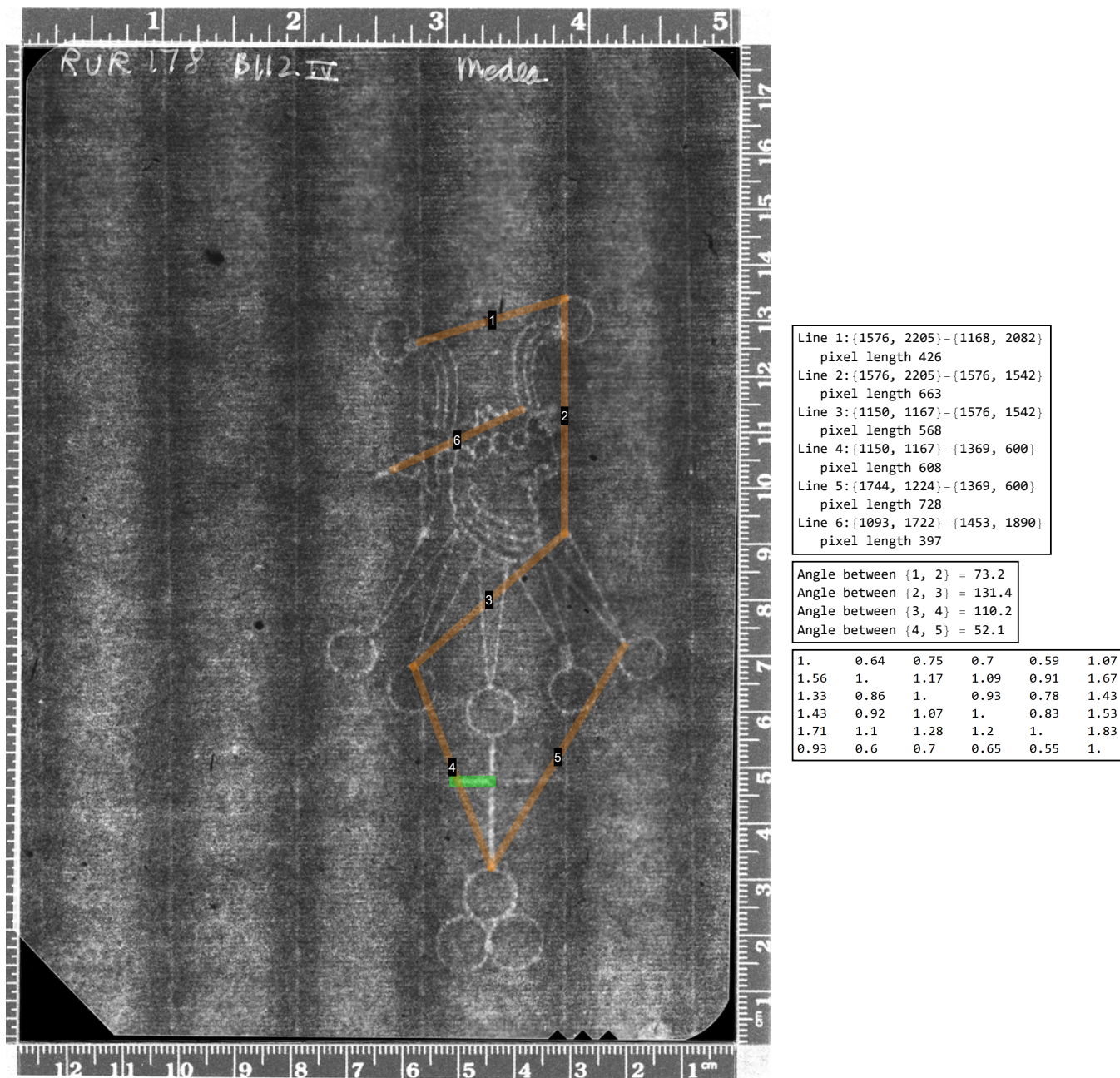


Figure 12: Green Line Beginning of a Line Segment to be Extended



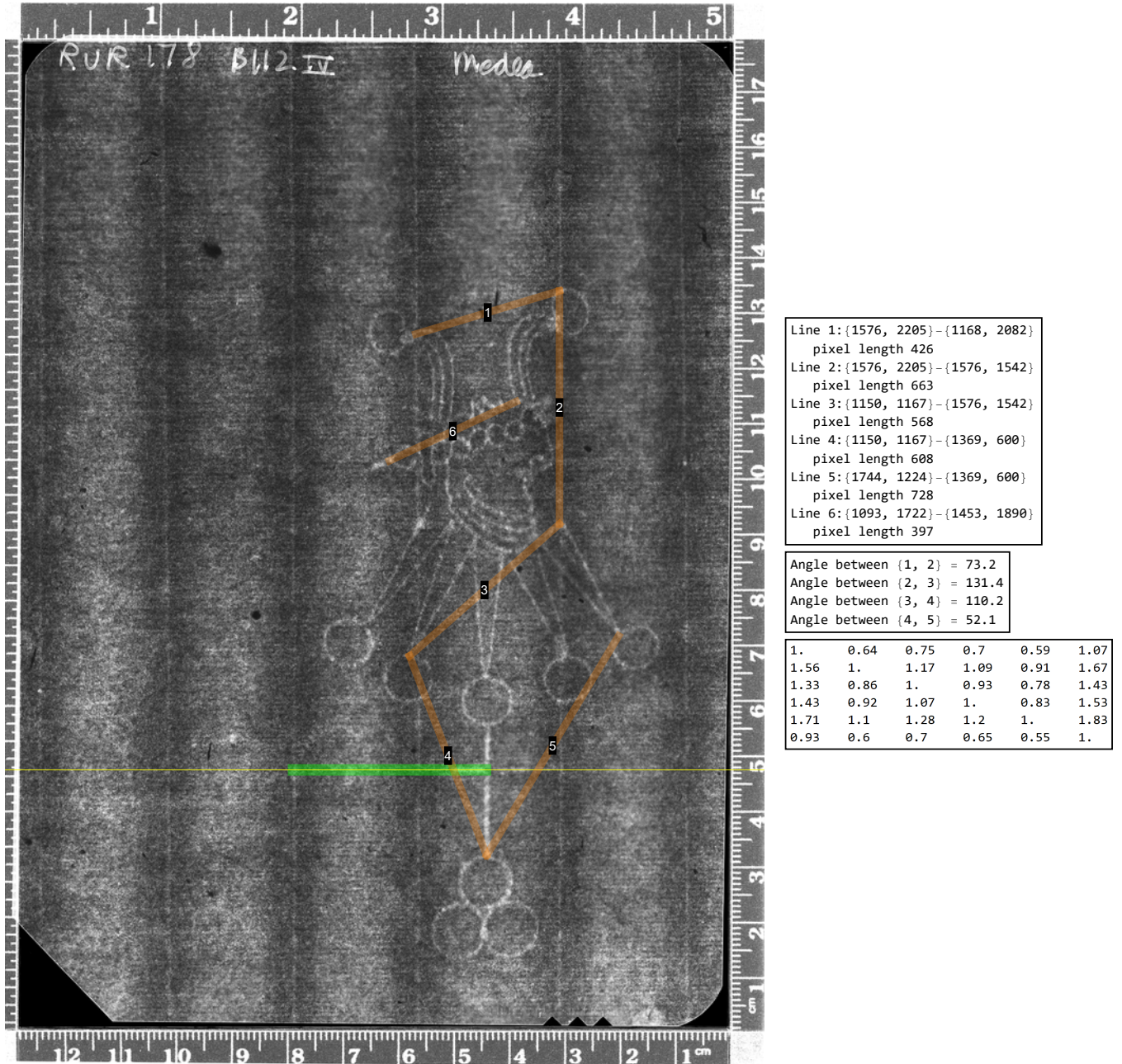


Figure 13: Extended Line with Relocated Left End

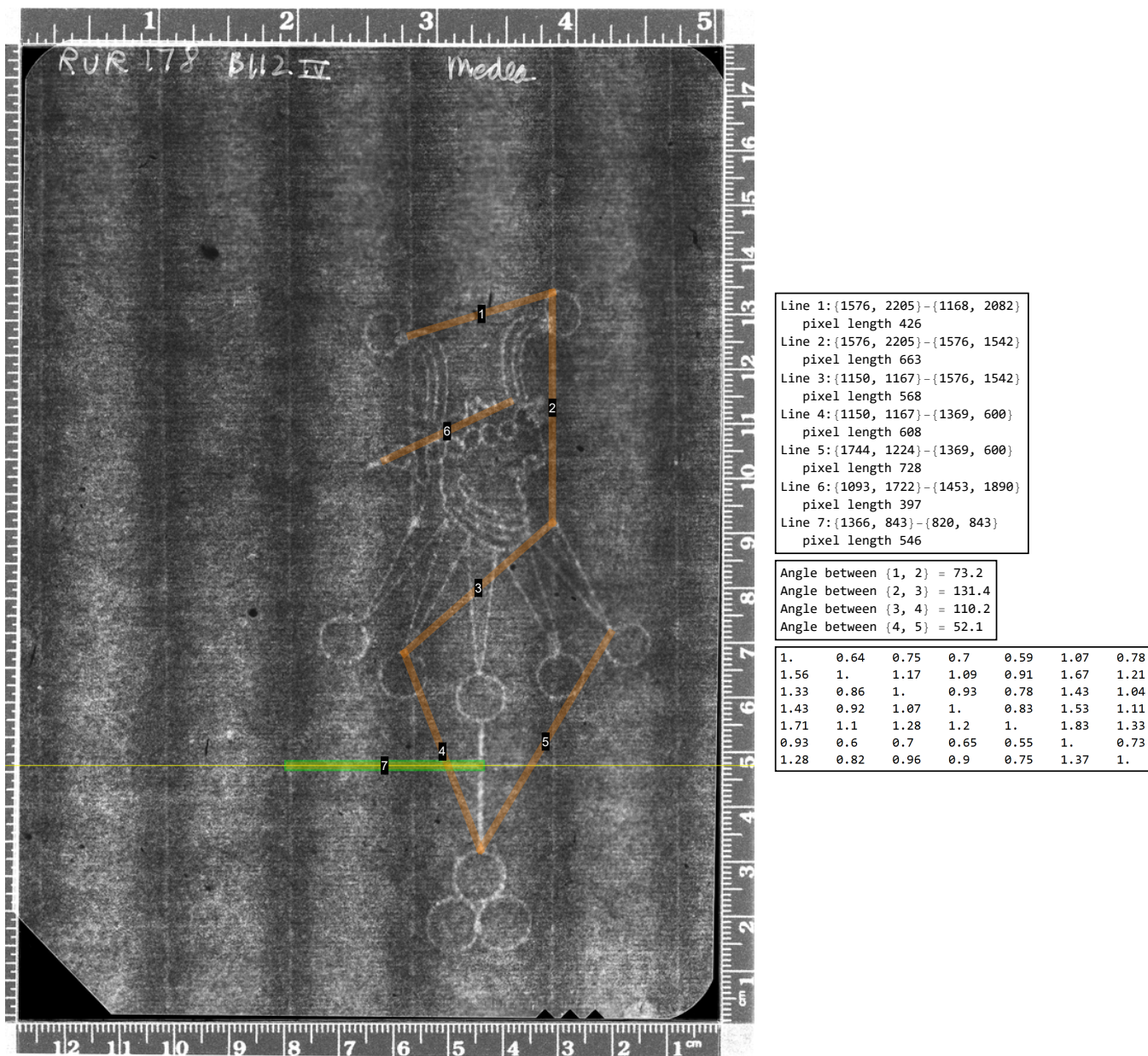


Figure 14: Image after Line 7 is Saved

## O. RATIO TABLE AND WATERMARK CODE

The  $i$ th row and  $j$ th column ratio-matrix entry is the computed length ratio with the length of line  $i$  divided by length of line  $j$ , as illustrated in the following table.

1/1	1/2	1/3	1/4	1/5	1/6	1/7
2/1	2/2	2/3	2/4	2/5	2/6	2/7
3/1	3/2	3/3	3/4	3/5	3/6	3/7
4/1	4/2	4/3	4/4	4/5	4/6	4/7
5/1	5/2	5/3	5/4	5/5	5/6	5/7
6/1	6/2	6/3	6/4	6/5	6/6	6/7
7/1	7/2	7/3	7/4	7/5	7/6	7/7

It is best not to mark very small lines or very long lines. The longest line should not be greater than a factor 4 times the shortest line. A selected set of these ratios can serve as an identifying code for the watermark. A recommended code is to choose the median length line as the numerator in each ratio so the values in the code are half below and half above 1.0. For example, in Figure 14 line 3 of 7 has the median length of 568 and the associated code of ratios with the length of line 3 as the numerator, i.e. 3/1, 3/2, 3/3, 3/4, 3/5, 3/6, 3/7, is provided by row 3 of the ratios matrix: 1.33, 0.86, 1.00, 0.93, 0.78, 1.43, 1.04 with 3 values below 1 and three above and all between 0.5 and 2.0.

## P. CLOSE MATCHING ASSESSMENT USES BOTH ANGLE AND RATIO DATA

Close matches of angles and ratios for similarly located lines are required for moldmate status (as are matching chainspace ratios aligned to the location of the watermark). A text file with the same snapshot name and number is provided of the data in the dimension, angle, and ratio tables each time a snapshot is taken. This information is provided in various formats for easier transcription.

## Q. CLOSING

When your use of `watermarkMarker` is complete and you wish to close it, select “Close” from the pull down “File” menu of the Wolfram Player. In response to the question asking if you want to “Save changes to `watermarkMarker`” click “Don’t save”.

# 5 A Case Study: Foolscape with Five-pointed Collar

We begin with the radiographs<sup>14</sup> of six Rembrandt prints in Figure 15. While all the watermarks in Figure 15 are Foolscape with Five-pointed Collar, they can be distinguished by the circular bells at their tops. In the first row of three radiographs in Figure 15 these bells are approximately level with each other. The radiographs in the second row all exhibit bells that are not level - with the bell on the side with the face noticeably higher.

There are other distinctions between the watermarks in the two rows, but we will begin by examining their chainspace codes. With the radiographs in the top row labeled 1, 2,

<sup>14</sup>The authors express their gratitude to the Dutch University Institute for Art History in Florence for providing access to their extensive collection of scans of radiographs of prints by Rembrandt. Five images in Figure 15 are from their collection: top row left to right - EH015025, EH015038, EH026023; bottom row left and middle - EH015040, EH0190053. The sixth in the bottom right corner is the radiograph from the Morgan Library & Museum used in the preceding “Guided Tour” sections.



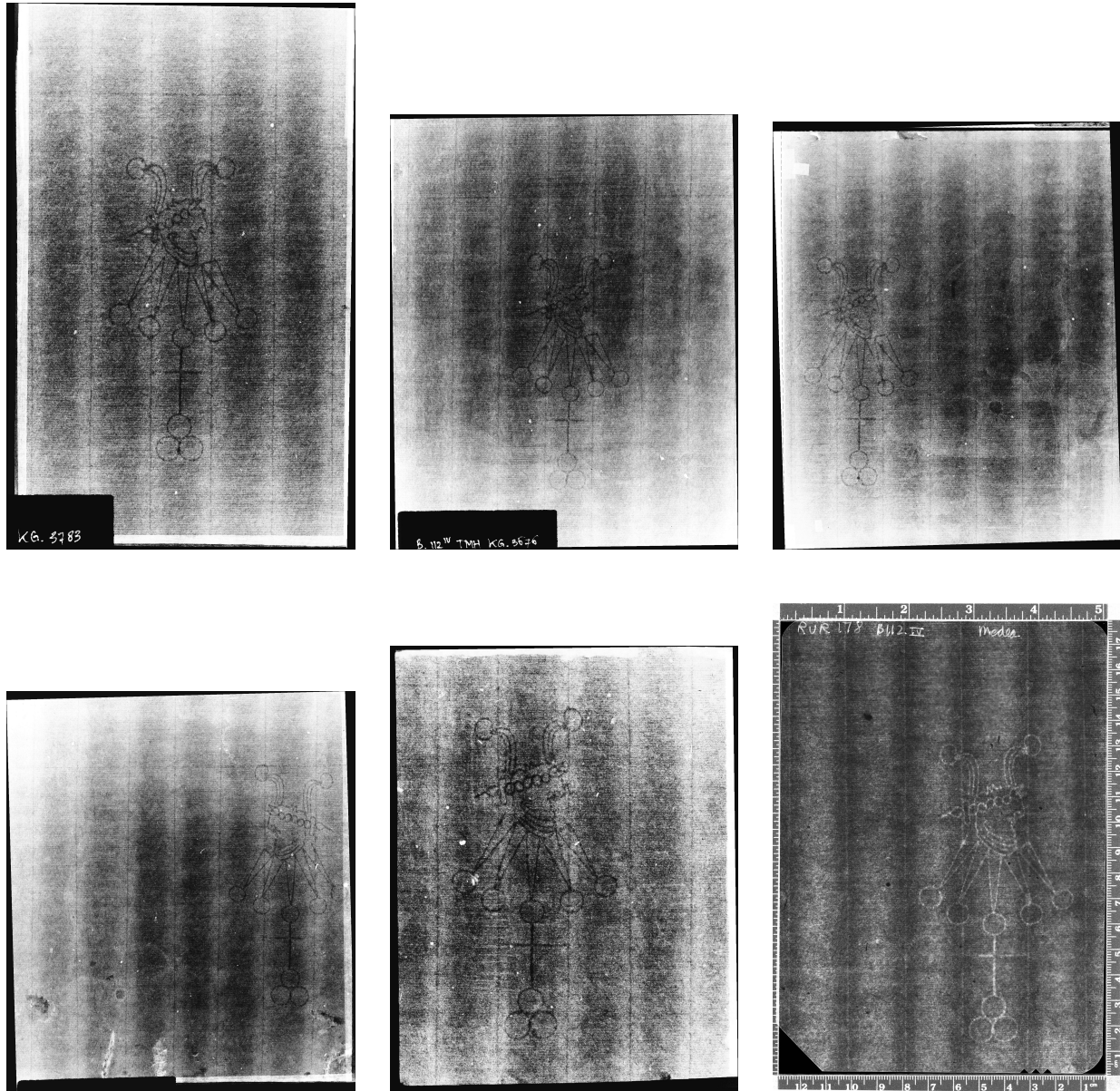


Figure 15: Six Radiographs Displaying Fooscap with Five-pointed Collar Watermarks

and 3 and the bottom row labeled 4, 5, and 6, the following table of ratios rounded to two decimal places (with all radiographs oriented before marking so the head is facing to the right) demonstrates that they form two separate groupings. All entries are aligned with the **ratio** indicating the pair having the space in which the watermark is centered as its right (denominator) value. (A dash was inserted for ratios not present in the respective images.) With a gap of over 0.07 between the ranges for any aligned ratios the two codes are not considered a match. This is especially apparent in the **ratio** including the watermark and the ratio in the table to its right.

Range 1-3	0.97	1.01-1.02	<b>0.97-0.99</b>	0.96-1.00	1.04-1.06	0.98	1.03	1.02
1	-	1.02	<b>0.97</b>	1.00	1.05	-	-	-
2	0.97	1.00	<b>0.99</b>	0.96	1.06	1.03	1.01	1.10
3	-	-	<b>0.97</b>	0.98	1.04	0.98	1.03	1.02
Range 4-6	0.99	1.02	<b>0.85-0.87</b>	1.13-1.15	0.99-1.01	0.94	0.92	1.18
4	-	-	-	1.13	1.01	0.94	0.92	1.18
5	-	-	<b>0.85</b>	1.15	0.99	-	-	-
6	0.99	1.02	<b>0.87</b>	1.15	-	-	-	-

The images within each of the two groups (images 1-3 and images 4-6) display tight matches, i.e. all aligned ratios have a range of less than 0.05. This is consistent with the two groups each being a group of three moldmates. Note that radiograph number 6 in Figure 15 is the Morgan Library & Museum radiograph used in the Guided Tour sections. The graphical visualization in Figure 16 of the ratio sequences emphasizes the tightness of the two groups and the distinction between the two groups.

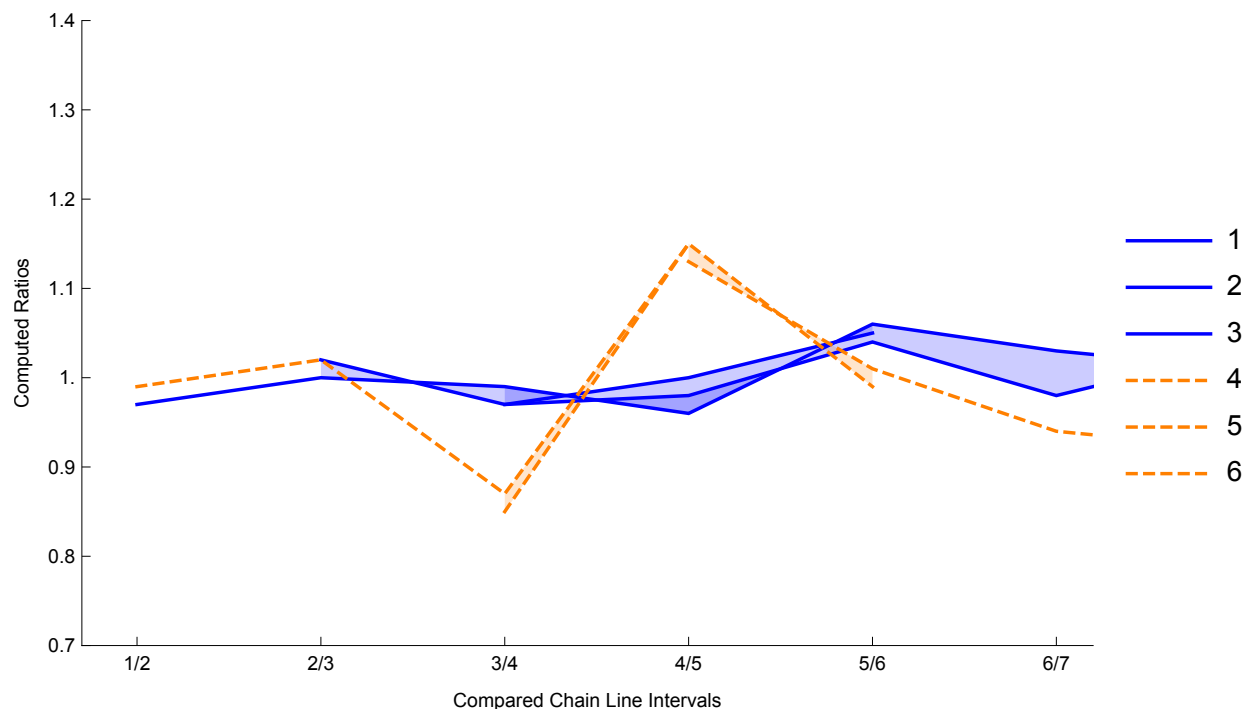
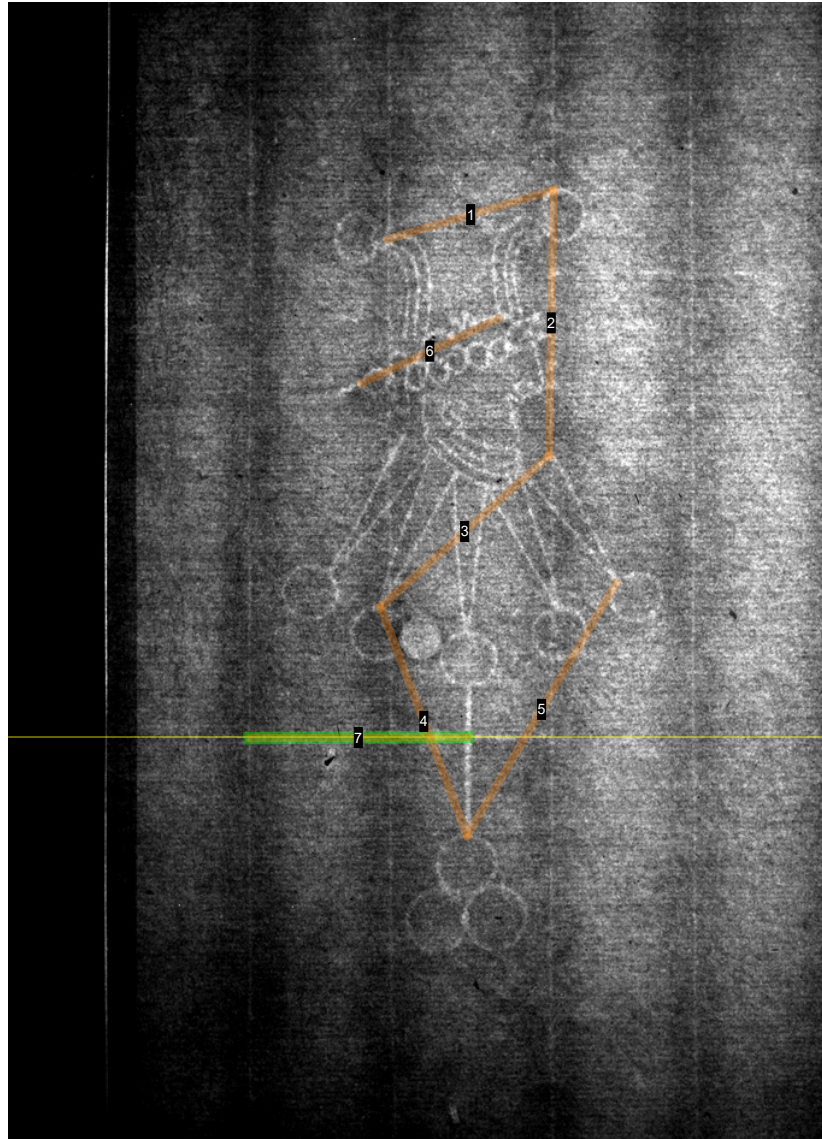


Figure 16: Chainspace Ratio Sequences



```

Line 1: {1360, 2370} - {946, 2247}
pixel length 432
Line 2: {1360, 2370} - {1348, 1707}
pixel length 663
Line 3: {928, 1329} - {1348, 1707}
pixel length 565
Line 4: {928, 1329} - {1144, 759}
pixel length 610
Line 5: {1516, 1389} - {1144, 759}
pixel length 732
Line 6: {880, 1887} - {1222, 2049}
pixel length 378
Line 7: {601, 1002} - {1144, 1005}
pixel length 543
    
```

```

Angle between {1, 2} = 72.4
Angle between {2, 3} = 133.
Angle between {3, 4} = 111.2
Angle between {4, 5} = 51.3
    
```

1.	0.65	0.76	0.71	0.59	1.14	0.8
1.54	1.	1.17	1.09	0.91	1.75	1.22
1.31	0.85	1.	0.93	0.77	1.49	1.04
1.41	0.92	1.08	1.	0.83	1.61	1.12
1.69	1.1	1.29	1.2	1.	1.93	1.35
0.88	0.57	0.67	0.62	0.52	1.	0.7
1.26	0.82	0.96	0.89	0.74	1.43	1.

Figure 17: Marked Foolscap with Five-pointed Collar K.a.b from Hinterding



To further identify the watermark in radiograph number 6, we now utilize `watermarkMarker`.<sup>15</sup> The third rows of the ratio matrices of Foolschap with Five-Pointed Collar K.a.b from Hinterding in Figure 17 and RVR178B from the Morgan Library & Museum in Figure 14 (which is the unmarked image 6 in Figure 15) are strikingly similar. The graphical visualization of the ratios in Figure 18 emphasizes the close fit of the two watermarks.

Hinterding	1.31	0.85	1.00	0.93	0.77	1.49	1.04
Morgan	1.33	0.86	1.00	0.93	0.78	1.43	1.04

The close match of the corresponding angles from their angle lists is apparent in the following table and visualized graphically in Figure 19.

Hinterding	72.4	133.0	111.2	51.3
Morgan	73.2	131.4	110.2	52.1

These tight matches confirm that image 6 in Figure 15 has a Foolschap with Five-pointed collar K.a.b watermark. The chain-space code matches for images 4, 5, and 6 in Figure 15 suggest that images 4 and 5 also have Foolschap with Five-pointed Collar K.a.b watermarks.

The designation of images 4, 5, and 6 as having Foolschap with Five-pointed Collar K.a.b watermarks, and images 1, 2, and 3 as having their twin watermark Foolschap with Five-pointed Collar K.a.a, can be confirmed with the use of the Foolschap with Five-pointed Collar decision tree in C. R. Johnson, Jr., “Decision Trees for Watermark Identification in Rembrandt’s Etchings,” to appear in the *Journal of Historians of Netherlandish Art*.

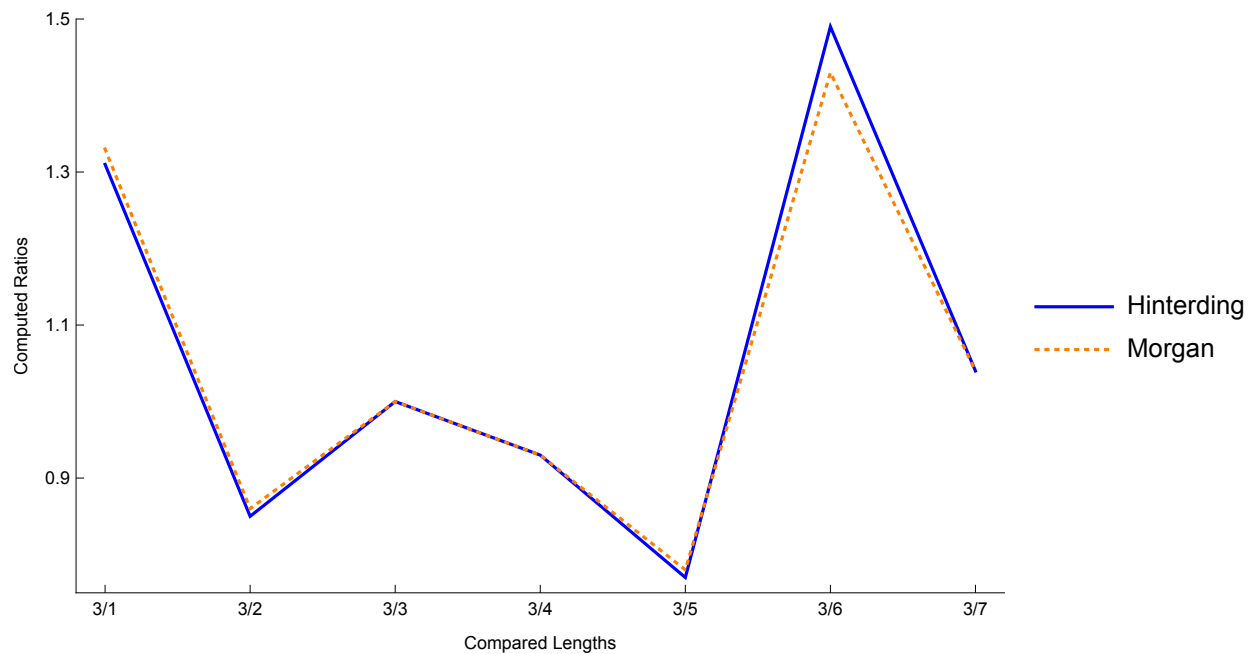


Figure 18: Watermark Line Segment Feature Ratios

<sup>15</sup>We will focus on just two Foolschap with Five-pointed Collar watermarks. One is the one from the Morgan Library & Museum used in the preceding “Guided Tour” sections. The other is taken from Erik Hinterding, *Rembrandt as an Etcher: The Practice of Production and Distribution*, 3 vols., Sound & Vision, 2006. The authors express their gratitude to Sound & Vision Publishers BV for providing permission to the WIRE Project at Cornell founded by C. R. Johnson, Jr. to use images from Hinterding’s book in their research and publications.

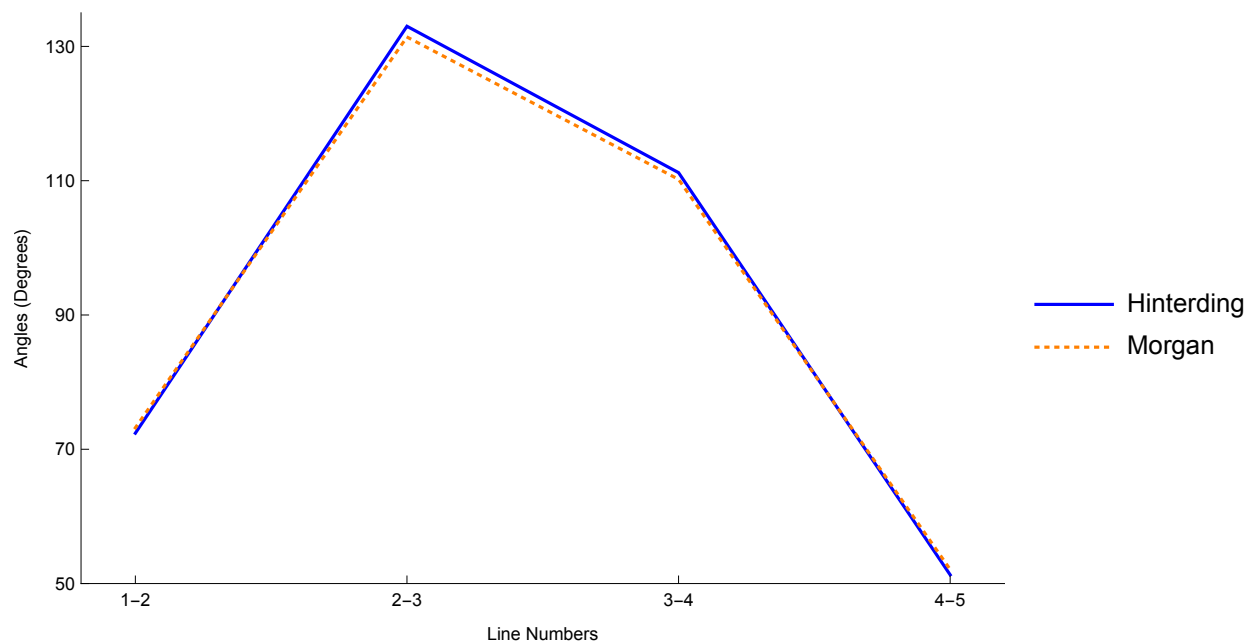


Figure 19: Watermark Connected Line Segment Angles

## 6 Publications Illustrating the Use of `chainLineMarker` and `watermarkMarker`

The moldmate verification tools `chainLineMarker` and `watermarkMarker` are also applicable for moldmate verification of watermarks not yet cataloged. In such cases the ratios and angles are computed for all watermarks under consideration and moldmate groupings are established with tight matches. The following references perform such an analysis for transmitted light images of papers from two of Leonardo’s codices and of pages from a 1536 French book.

- Margaret H. Ellis, William A. Sethares, and C. Richard Johnson, Jr., “Coding Leonardo’s Papers: Discovering Moldmates in the *Codex Arundel*,” submitted to *Master Drawings*, February 2020.

From the introduction: [W]e propose to characterize the watermarks and chain line/wire intervals of a selection of papers found in the *Codex Arundel* through the application of computational processes that enhance, measure, compare, and match them. These procedures, which rely on more generally available visible and transmitted light images, have two benefits for Leonardo scholars: 1. substantially improved watermark and chain line/wire pattern images, which are useful for the general characterization of Italian 16th century papers; 2. the detection of paper moldmates, that is, sheets formed from the exact same papermaking mold. The existence of moldmates within the *Codex Arundel* or, by extension, shared across other Leonardo codices, raises tantalizing questions vis-à-vis dating, sequence, studio practice, individual preferences, and periods of intense activity, as well as patterns of European paper production and distribution in the 15th and 16th centuries.

- Sara F. Gorske, C. Richard Johnson, Jr., William A. Sethares, Margaret Holben Ellis, and Paul Messier “Moldmate identification in 16th-century European paper using quantitative analysis of watermarks, chain line intervals, and laid line density”, submitted to *International Journal for Digital Art History*, February 2020.

Abstract: Antique laid paper, made in Europe in the 16th century, has the important quality that each sheet of paper made on the same papermaking mold retains a nearly identical imprint of the molds wire structure. These “moldmates” are identified by analyzing the recorded wire features, which are visible under transmitted light. When visual analysis is not sufficient to distinguish moldmates, three features of the molds wire mesh can be quantitatively analyzed using image processing techniques: watermark shape and placement, chain line intervals, and laid line density, for which a new method of analysis is introduced here. Using signal processing procedures, the frequency of the laid lines across a sheet of paper was found to fluctuate in a pattern unique to that mold. These quantitative methods were tested on a sample set of blank sheets from a 1536 edition of *De re militari* by Vegetius; computational analysis using any one of the three features was able to distinguish to which of four molds a sheet belonged. These results demonstrate that any of these techniques can be chosen as appropriate to determine moldmates from within a set of laid paper, regardless of size or the inclusion of a full watermark.

## 7 Acknowledgements

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