

# AUTOMATING OVERLAY VIDEOS

## USER'S GUIDE for `watermarkPointMarker` and `visualizeOverlays`

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### 1 Summary

This document introduces the current versions<sup>1</sup> of two software tools - `watermarkPointMarker` and `visualizeOverlays` - that produce an overlay video of two similar watermarks. These overlays are the preferred tool for comparing two watermarks, whether the goal is identifying distinguishing features between two similar watermarks or establishing that two watermarks are identical.

The program `watermarkPointMarker` uses mouse-driven placement of a sequence of points in a watermark. For comparison with another watermark marked at the same points, the points selected should be at precisely locatable points that span the height and breadth of the watermarks. Examples of precisely locatable points include places where two wires in the watermark cross or a chain line crosses a line in the watermark. Sharp points or corners in the watermark are also good candidates for point marking.

The purpose of the set of marked points is to provide a set of tie points guiding alignment performed by `visualizeOverlays`. The points must be marked in the same sequence for watermarks to be properly aligned for comparison. The number of marked points needs to be more than five. Due to the optimization routine used in `visualizeOverlays` to perform alignment, the more points that are marked the less the error will be in achieving visually acceptable placement of two matching watermarks.

The program `visualizeOverlays` takes the data and image files produced by `watermarkPointMarker` for a pair of watermark images and creates the overlay animation. The program manipulates one image by adjusting its

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<sup>1</sup>as Computable Document Format (.cdf) files programmed in Mathematica by William A. Sethares for use with Mathematica's free Wolfram Player, which can be downloaded from <https://www.wolfram.com/player/>. Use version 12.1.1 of the player. Do not use version 12.2.0, which has a bug.

scaling, rotation, and translation so as to reduce the (sum of the squares) distance between like-numbered points. This factor adjustment procedure continues until the best possible fit is achieved. The two watermarks do not need to be to the same scale or have the same resolution. The rotational orientations of the two marked watermarks do not have to be identical. For example, the chain lines in the two images do not both need to be exactly vertical. The two marked watermarks do need to be the same orientation in terms of a right-left or up-down flip, which cannot be corrected by simple rotation. For example, consider watermarks that are both single-headed eagles. They should both be marked after making certain that the eagle's beak points in the same direction (right or left) if the watermark were to be rotated so the eagle's head is at the top of the image. If this is not the case, it can be accomplished by a right-left flip of one of the watermarks before marking them.

Appropriate marking of the two watermarks with `watermarkPointMarker` followed by specifying the two marked watermarks for subsequent processing by `visualizeOverlays` produces a looping video. The video begins with one watermark which fades as the other watermark slowly appears. The video continues as the original watermark reappears and the other fades. Then this loop will repeat.

As an example, consider the print and drawing and their respective watermarks in Figure 1. The top row in Figure 1 presents the print *Christ Crucified Between Two Thieves: 'The Three Crosses'* by Rembrandt van Rijn and a radiograph revealing the the Strasbourg Bend watermark in its laid paper support. The bottom row in Figure 1 shows the drawing *Solomon Offering a Sacrifice of Peace Offerings* by Ferdinand Bol and a radiograph revealing the Strasbourg Bend watermark in its laid paper support. The marked versions of the two watermarks produced by `watermarkPointMarker` are shown in Figure 2. The seventeen points chosen for marking are numbered. The looping overlay video created by `visualizeOverlays`, which can be accessed by clicking on this [link](#), demonstrates that the two watermarks are an exact match as the video appears to be displaying a single watermark even though labels near the bottom and top of the image indicate which of the two watermarks are displayed.

The remainder of this document provides a tutorial introduction to the (present) capabilities of these tools with the examination of watermarks re-

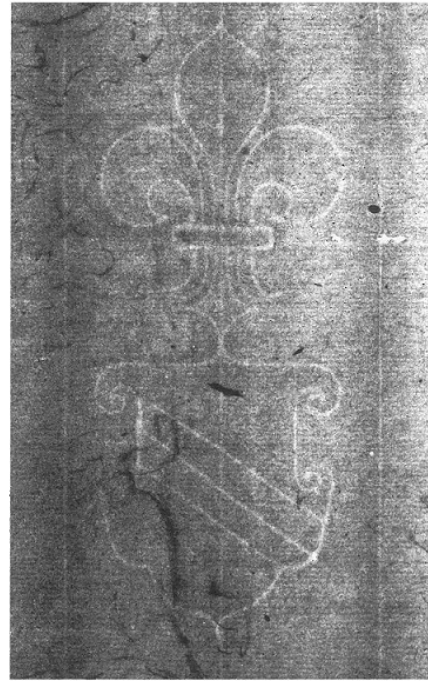


Figure 1: Upper Row: Rembrandt van Rijn, *Christ Crucified Between Two Thieves: 'The Three Crosses'*, B78iii :: Left: Metropolitan Museum of Art, 41-1-32; Right: Radiograph of Watermark; Lower Row: Ferdinand Bol, *Solomon Offering a Sacrifice of Peace Offerings* (Images from T. Vignau-Wilberg, *Rembrandt auf Papier: Werk und Wirkung*, Munich, 2001 ©Staatliche Graphische Sammlung München) :: Left: Graphische Sammlung München, 1744; Right: Radiograph of Watermark

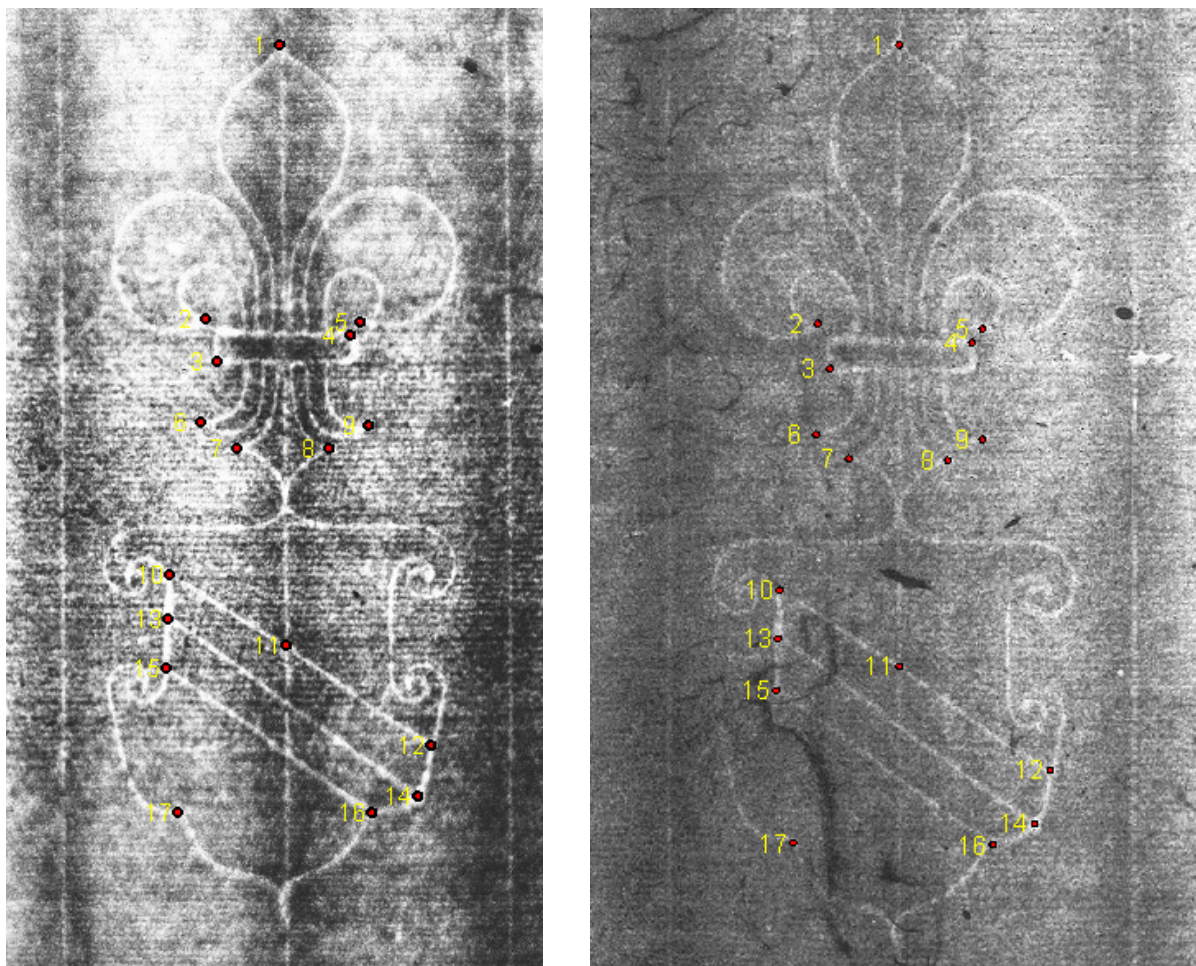


Figure 2: Marked Strasbourg Bend Watermarks :: Left: Rembrandt, Right: Bol

vealed in radiographs of Rembrandt’s prints as a sequence of operations each followed by observation of their results. These tools were designed so the technical expertise needed to successfully utilize them does not exceed that required to utilize Photoshop.

## 2 Guided Tour: watermarkPointMarker

Presuming previous installation of the free Wolfram CDF player, you are to begin with the following files<sup>2</sup> in a folder on your computer desktop:

- watermarkPointMarker07.cdf
- 41-1-32dpi600.jpg

The following sequence of operations introduces `watermarkPointMarker`, which will allow you to designate a number of precisely selectable mouse-clicked points in the image of the watermark. The software will provide an image with marked points and a table of the pixel coordinates of the marked points intended for use in `visualizeOverlays`.

### A. OPEN PLAYER

Open free Wolfram CDF Player. Choose “Open” from the file menu and open `watermarkPointMarker07.cdf`.

### B. ENABLE DYNAMICS

Click “Enable Dynamics” button in upper right. A default image will appear as shown in Figure 3.<sup>3</sup>

### C. SIZE IMAGE

To allow the largest viewing area, it is recommended that you go to full screen mode.

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<sup>2</sup>The authors express their gratitude to the Metropolitan Museum of Art for permitting the use of their scan of the radiograph 41-1-32dpi600.jpg of their print by Rembrandt va Rijn, *Christ Crucified Between Two Thieves: ‘The Three Crosses’*, B78iii, 41-1-32, 1653.

<sup>3</sup>The authors are grateful for the permission granted by the Bill and Melinda Gates Collection to use an Eagle watermark from the Leicester codex as the default image. This default image is acquired from processing of a transmitted light image as described in W. A. Sethares, M. H. Ellis, and C. R. Johnson, Jr., “[Computational Watermark Enhancement in Leonardo’s Codex Leicester](#),” *Journal of American Institute of Conservation*, vol. 59, issue 2, pp. 87-96, March 2020.

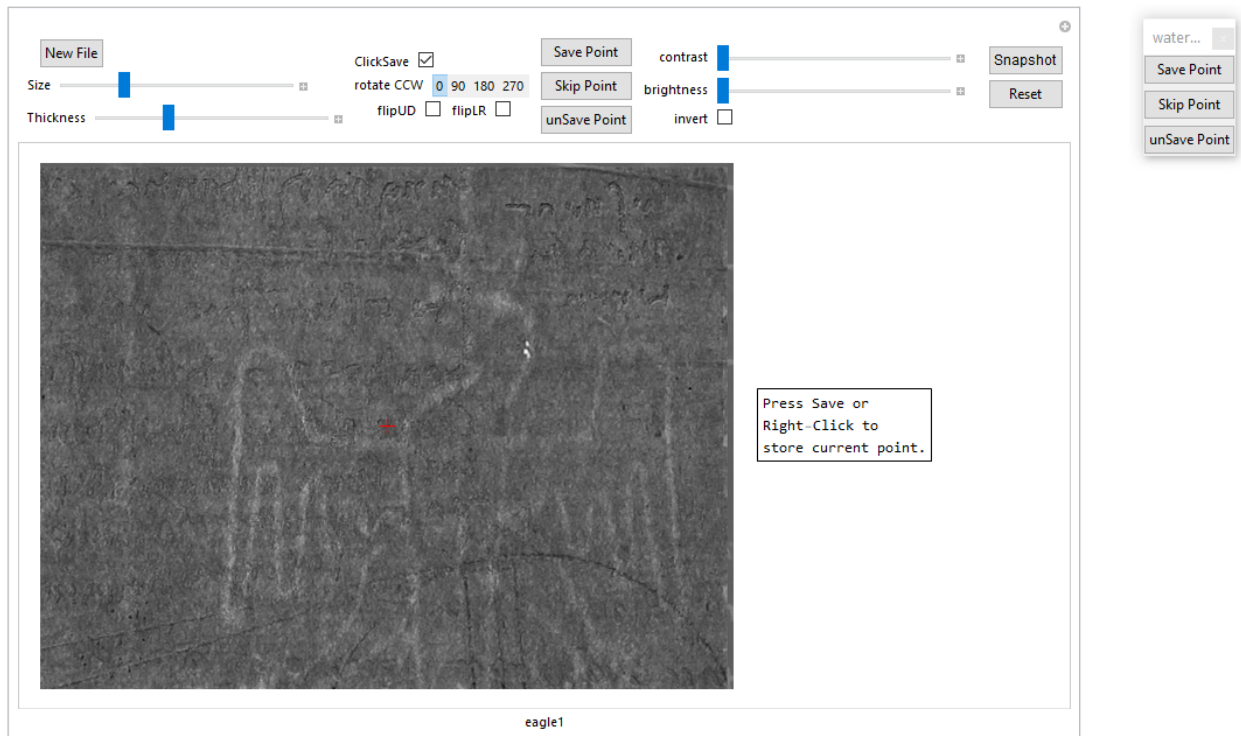


Figure 3: Start Screen for watermarkPointMarker

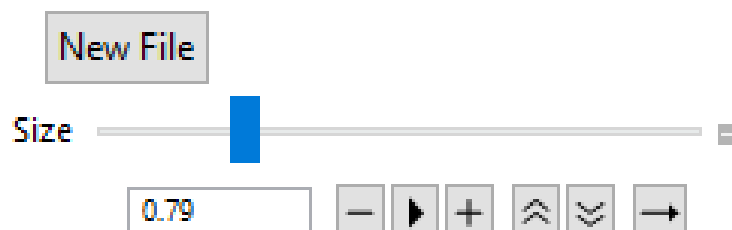


Figure 4: Size Slider and Control Buttons

- ⊕ *Size Slider* :: You can use the “size” slider below the “New File” button to zoom in on the displayed image. Moving the slider to the right increases the dimensions of the image. Correspondingly, moving it to the left will decrease the dimensions of the image. The full range for this size multiplier is 0.1 to 3.
- ⊕ *Multiplier in a Box* :: If you prefer to scale the image dimensions by a specific numerical value, first click the plus at the end of the slider illustrated in the close-up in Figure 4. This will reveal a box with the chosen multiplier. Moving the slider will adjust the multiplier displayed in this box. The multiplier can be set by entering a specific value from 0.1 to 3.
- ⊕ *Incremental Multiplier Changes* :: Figure 4 also reveals 6 buttons activated by pressing the plus at the right end of the size slider. From left to right, the first button when clicked reduces the size multiplier by 0.2. The third button increases the size multiplier by 0.2.
- ⊕ *Animated Multiplier* :: The second button of the six to the right of the multiplier box controls play and pause options for viewing the effects of a smoothly moving slider. Pressing the play option (an arrowhead pointing to the right) moves the slider until the second button is pressed again converting it to the pause symbol (a pair of parallel vertical lines). The direction of the motion is dictated by the sixth button. It has three choices.

An arrow pointing right results in the slider moving to the right until it reaches the maximum setting (of 3), resets itself to the minimum value (of 0.1) and starts moving to the right again. It will continue until the second button is pressed to pause the slider’s motion.

Clicking the 6th button can reveal an arrow pointing left, which when the second button is set to play causes the slider to move to the left to the minimum setting, jump to the maximum setting, move to the left to the minimum, and repeat until stopped with the second (play/pause) button.

The third choice for the sixth button is a line with an arrowhead at each end with the one at the right end pointing to the right and at the left end pointing to the left. This results in a back and forth motion from one end of the slider bar to the other.

The fourth and fifth slider control buttons increase (double up arrow) or decrease (double down arrow) the speed at which the slider moves.

A reasonable image size choice for watermark point marking is a setting that allows viewing of just the region around the point to be marked. For large images exceeding the size of your computer screen, sliders on the right and bottom sides of the image window can be used to position the view seen on the screen to the desired portion of the full image.

#### D. ROTATE IMAGE

The string of numbered buttons to the right of the Size slider (See Figure 3) labelled with “rotate CCW” can be used to rotate the image in the counterclockwise (CCW) direction. The “90”, “180”, and “270” buttons rotate the original image counterclockwise by 90, 180, and 270 degrees, respectively. The “0” setting returns the image to its original orientation.

#### E. FLIP IMAGE

Further image reorientation possibilities are provided by the options below the bar of rotation choices. Clicking the box to the right of flipUD flips the image up-down. The top of the image is pulled forward and down so that the two-dimensional image is now shown upside down from the back. (Note that this differs from a 180 degree rotation, which retains the side being viewed prior to rotation.) Clicking the box to the right of flipLR similarly flips the image left-right. (Note that flipUD followed by flipLR is the same as a 180 degree rotation.) These reorientation functions can be used to obtain a desired watermark orientation.

#### F. CONTRAST AND BRIGHTNESS ADJUSTMENT AND PIXEL VALUE INVERSION

To improve the contrast or brightness of the image to make discernment of the watermark more precise, you can use the sliders to the right of the column of Save Point, Skip Point, and UnSave Point operators (See Figure 3). If you wish to invert the black pixels and white pixels to white and black for more comfortable viewing, just click the box to the right of the invert label situated below the brightness and contrast sliders. If your image is in color, you may be surprised by the appearance of the inverse colors.



## G. OPEN NEW IMAGE FILE

Click “New File” button in upper left. Navigate to the file containing the image to be marked - in this example 41-1-32dpi600.jpg - and double click on it. watermarkPointMarker can access only .tif or .jpg files. Manipulate the size so the entire image of 41-1-32dpi600.jpg is visible on your screen, as displayed in Figure 5.

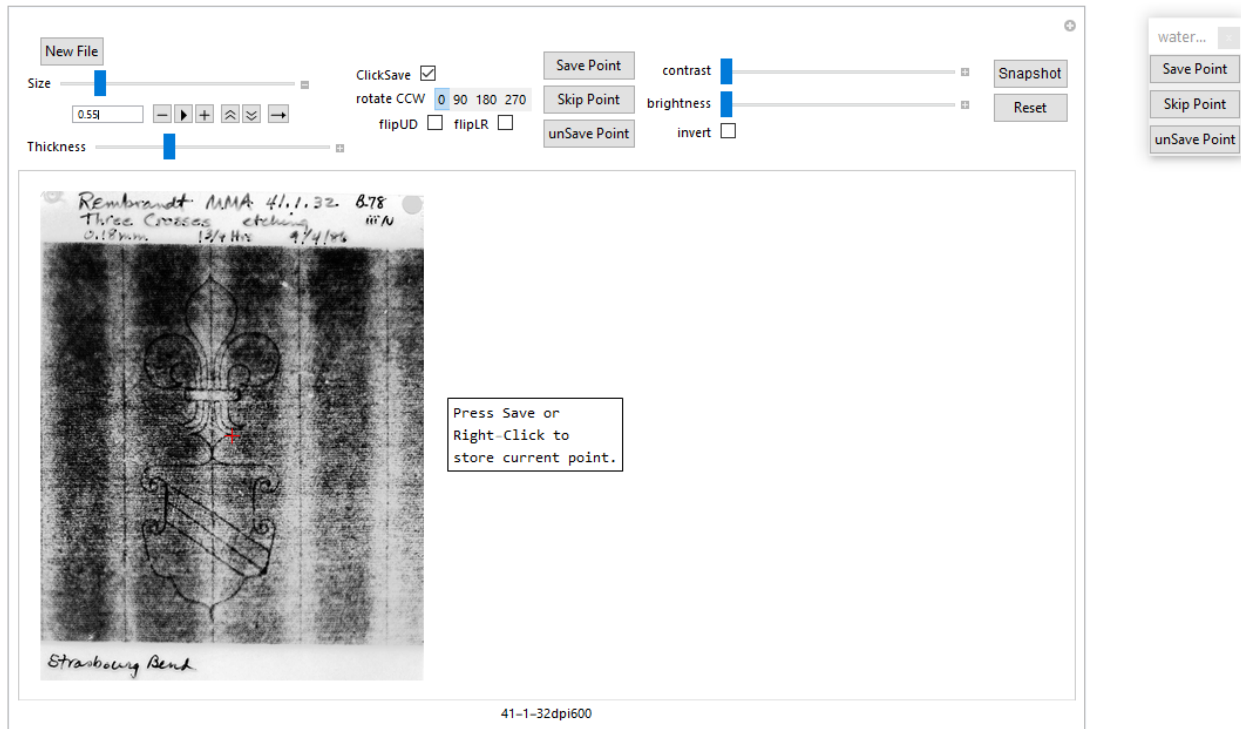


Figure 5: After Selection of 41-1-32dpi600.jpg as the New File

## H. MARKING A POINT

When a New File is selected a red “+” will appear in the center of the image, as can be seen in Figures 3 and 5. This red “+” needs to be moved to the desired location of the first (number 1) point to be marked, which is the top point of the fleur-de-lis.

To relocate the red “+” use the mouse to move its pointer inside the displayed image where it will become an orange “x”. Continue to use the mouse to move the orange “x” to the desired location and click the mouse. This will relocate the red “x” to the clicked location. The center of the red “+” will be placed at the center of the orange “x”. If you are not satisfied with the achieved location of the red “+”, you can move

the orange “x” and click again, which will move the red “+” to this new location. Figure 6 illustrates the result of marking the point at the top of the fleur-de-lis.

## I. THICKNESS SLIDER

Specific point marking can be more precise when using an enlarged image. Use the size slider to enlarge the image so its width extends across the width of your screen. This maneuver may reveal that the red “+” is not exactly located at the tip of the top of the Fleur-de-Lis. Relocate the red “+” if necessary. It might be convenient to move the red “+” away from the location you wish to mark in order to more clearly see exactly where you want the center of the orange “x” to be set and clicked.

You will notice that the red “+” did not change size when the image was enlarged with the size slider. When the full image is captured this will cause the red “+” to appear proportionately smaller. This can be corrected with the Thickness Slider which is below the size slider. Moving the Thickness Slider to the right increases the size of the red “x” (and all subsequent red “x”s if the Thickness Slider is left at this setting). Moving the Thickness Slider to the left decreases the size of the red “x”. Clicking the plus at the end of the Thickness Slider reveals a row of options for manipulating the Thickness Slider setting that operates just like the matching bar that accompanies the Size Slider as illustrated in Figure 4 with its options described in section C.

Enlarging the image in Figure 6, re-positioning the red “+”, and utilizing the Thickness Slider results in Figure 7.

## J. SAVING A POINT

There are three ways to save a point and then move on to selecting and saving the next point. One is to click the Save Point button at the top of the column of Save Point, Skip Point, and UnSave Point operators to the left of the Contrast and Brightness Sliders. Another is to click the Save Point button in the column of similar operators in the movable “water...” palette. When working with an image that is larger than your screen, it can be inconvenient to have to return to the top of the image to click the Save Point button next to the Contrast Slider. While the palette can be moved by clicking on its top white strip and dragging

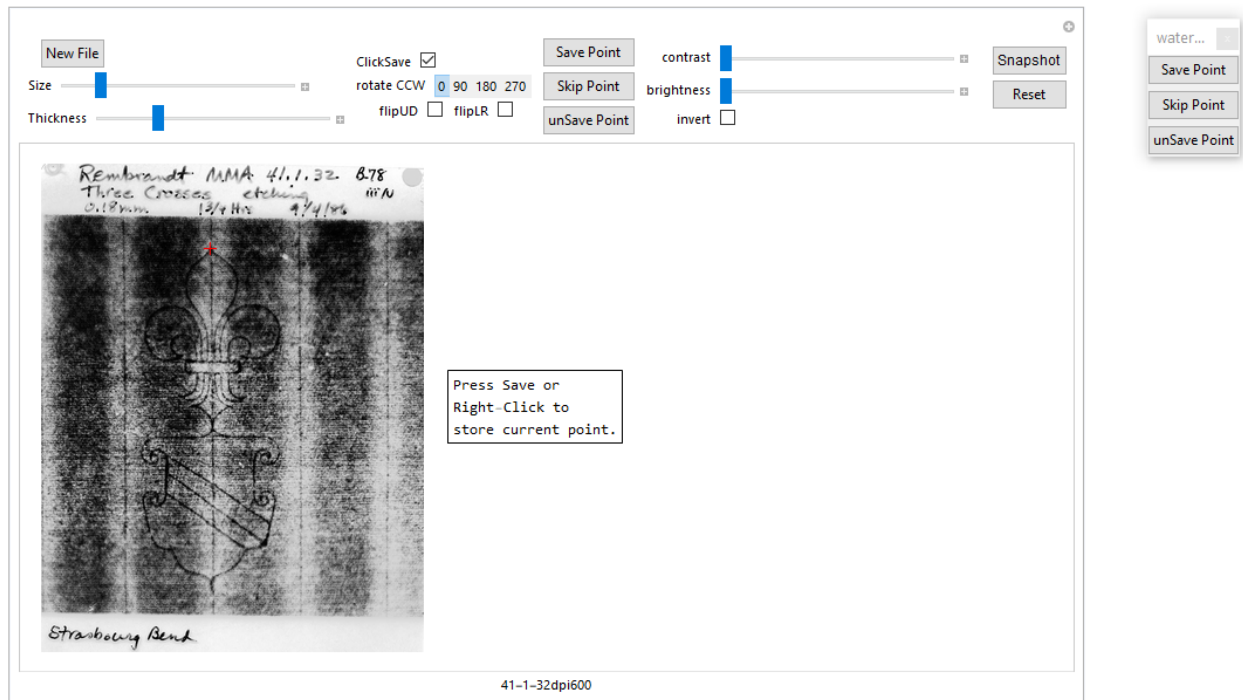


Figure 6: After Marking the First Point at the Top of the Fleur-de-Lis

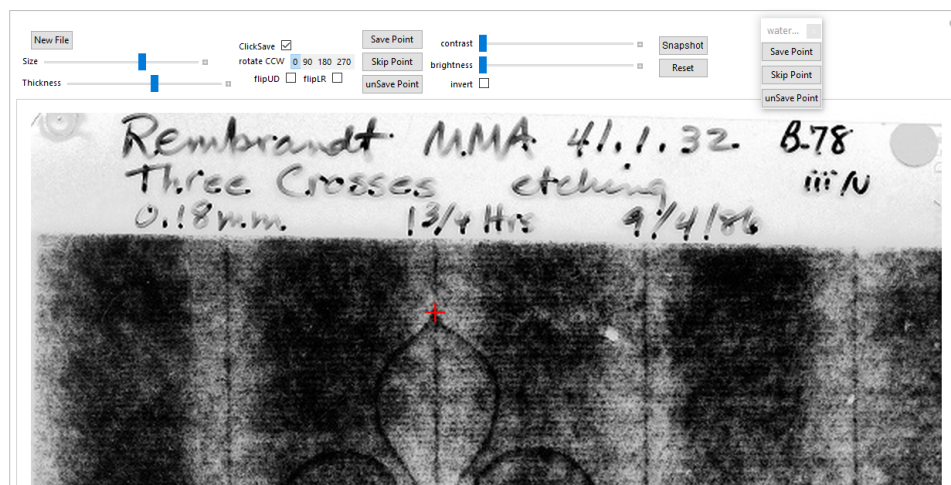


Figure 7: After Image Enlargement and Point 1 Re-positioning and Thickness Adjustment

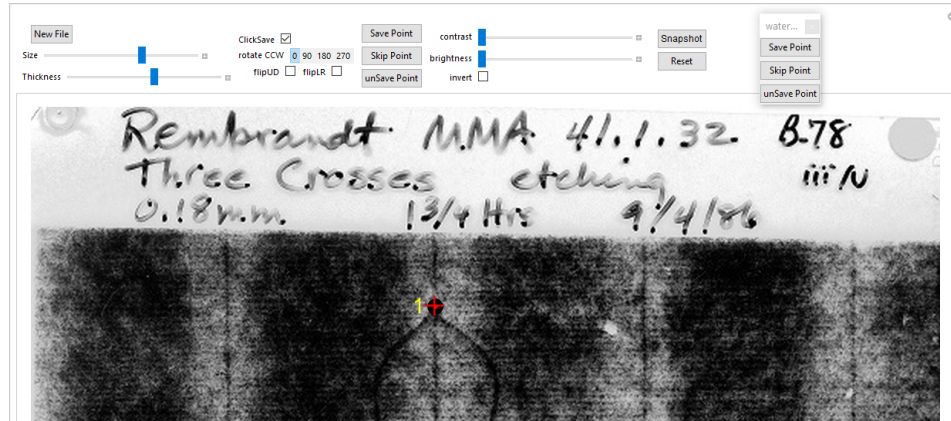


Figure 8: Saved Numbered Point

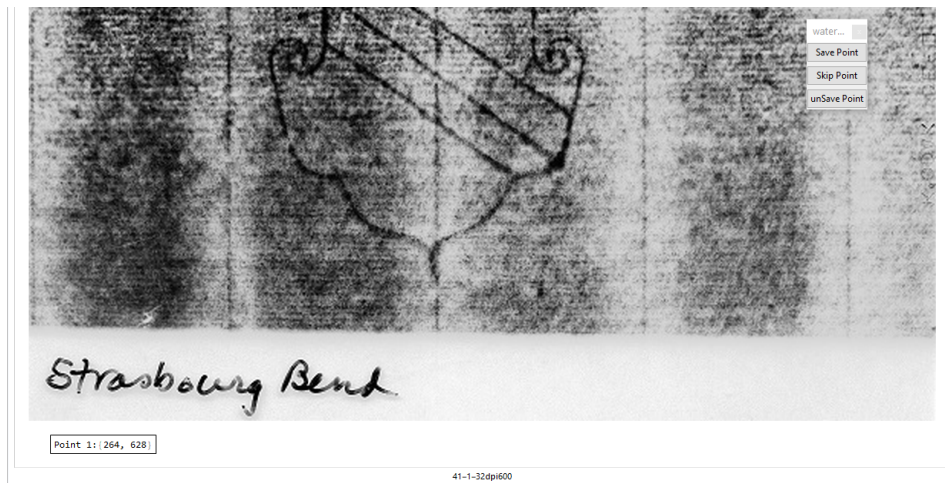


Figure 9: Saved Point 1 Location Coordinates

it, once it is at a particular location within the screen it does not move as the image is relocated with the sliders to the right and below the image. The third option does not require relocating the palette. If the ClickSave box to the left of the Save Point button is checked (as it is in Figure 7) right-clicking the mouse will save the point where the red “+” is located. If the ClickSave box is not checked, clicking the right mouse button does nothing.

A screen shot with the saved marked spot as in Figure 8 shows that it numbers the marked spot and marks it with a black bull’s eye with a red center. Also, it adds an entry to the table of coordinates (in pixels) of each of the marked points. Figure 9 shows the table added at the bottom of the image after saving point 1.

If a point marked in one of the two images to be compared is not visible in the other image because it is obscured or it is not included because the associated print only has a fragment of a watermark in its laid paper support, the Skip Point button below the Save Point button must be used so all of the visible marked points in the two images are labelled with the same numbers.

If for any reason the numbering goes awry, the unSave button below the Skip Point button can be clicked a sufficient number of times to remove all of the incorrectly numbered points starting with the one with the highest number and continuing with the point among those remaining with the highest number. Then their marking must be repeated with the correct numbering. Points that are unSaved will also be removed from the table of point locations.

## K. TAKING A SNAPSHOT

To save a copy of the current workspace at any time press the Snapshot button to the right of the brightness and contrast sliders.<sup>4</sup> The snapshot will appear as a pdf file in the format filenameSnapWM-year-month-day-hour-minute-second.pdf in the folder containing `watermarkPointMarker`. Figure 10 shows the snapshot of the watermark with the one marked location (with the red “+” in Figure 7 removed and replaced by a numbered black bull’s eye with a red center) and to its right a table of the coordinates of the marked location. Invoking the snapshot button will

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<sup>4</sup>One exception is the movable “water...” palette, which will not appear in a snapshot.



Figure 10: Strasbourg Bend Watermark of 41-1-32 with a Single Mark and a Side Table with the Marked Point's Coordinates

also provide a .jpg file of just the watermark with the same name except for the “WM” replaced by “WMImg” and the same time stamp. A .txt file will also be created of just the coordinates with the same name except for the “WM” replaced by “TextWM” along with the same time stamp. This .txt file will present the data in various formats as an aid to transcription of the data into a report.

The pdf file (41-1-32dpi600SnapWM2020-11-15-15-59-00.pdf) from the example in the Summary section for the watermark in the print from the Metropolitan Museum of Art with 17 marked points - appearing (without the associated table of marked point coordinates) in Figure 2 - is presented in Figure 11. The watermark in Figure 11 has white wires while the same watermark in the preceding figures of this Guided Tour section has black wires as in the original image provided by the Metropolitan Museum of Art. The conversion was made by checking the invert box mentioned in step F because the overlays are clearer in this case with white wires. The watermark as it appears - greyscale inverted - before marks are added is the jpg file (41-1-32dpi600SnapWMImg2020-11-15-15-59-00.jpg) produced by `watermarkPointMarker`. It is the same as the image in Figure 11 but without the marked and numbered points. The text file (41-1-32dpi600TextWM2020-11-15-15-59-00) documenting the coordinates of the marked points is

```
{{262.4, 627.}, {217.4, 460.}, {224.4, 434.}, {305.4, 450.},
{311.4, 458.}, {214.4, 397.}, {236.4, 381.}, {292.4, 381.},
{316.4, 395.}, {195.4, 304.}, {266.4, 261.}, {354.4, 200.},
{194.4, 277.}, {346.4, 169.}, {193.4, 247.}, {318.4, 159.},
{200.4, 159.}}
```

```
Point 1:{262, 627}
Point 2:{217, 460}
Point 3:{224, 434}
Point 4:{305, 450}
Point 5:{311, 458}
Point 6:{214, 397}
Point 7:{236, 381}
Point 8:{292, 381}
Point 9:{316, 395}
Point 10:{195, 304}
Point 11:{266, 261}
```

```

Point 12:{354, 200}
Point 13:{194, 277}
Point 14:{346, 169}
Point 15:{193, 247}
Point 16:{318, 159}
Point 17:{200, 159}

```

## L. Reset

If you want to re-start the marking process at step H, use the Reset button below the Snapshot button. Pressing the Reset button will clear all marked points and the table of marked point locations. All other settings for size, thickness, invert, rotation, flip, brightness, and contrast will be retained.

## M. CLOSING

When your use of `watermarkPointMarker` is complete and you wish to close it, select “Close” from the pull down “File” menu of the Wolfram Player. In response to the question asking if you want to “Save changes to `watermarkPointMarker`” click “Don’t save”.

## 3 Guided Tour: visualizeOverlays

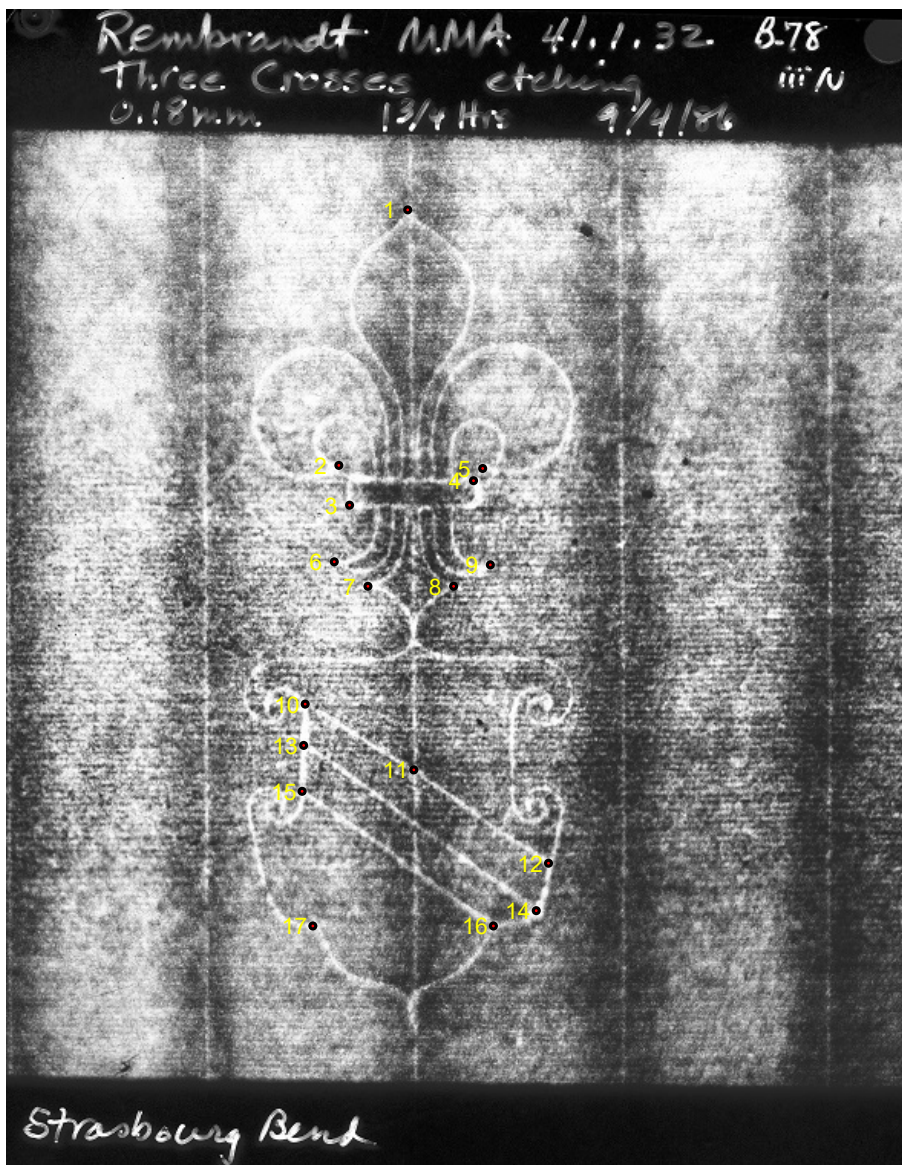
Presuming previous installation of the free Wolfram CDF player, you are to begin with the following files in a folder on your computer desktop resulting from prior application of `watermarkPointMarker` to radiographs of the watermarks in two prints:<sup>5</sup>

- `visualizeOverlays9.cdf`
- `41-1-32dpi600SnapWM2020-11-15-15-59-00.pdf`
- `41-1-32dpi600SnapWMImg2020-11-15-15-59-00.jpg`
- `41-1-32dpi600TextWM2020-11-15-15-59-00.txt`
- `RVR122B78iiBetadpi400SnapWM2020-11-15-16-06-41.pdf`

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<sup>5</sup>The authors express their gratitude to the Morgan Library & Museum for permitting the use of their scan of the beta-radiograph RVR122 of their print by Rembrandt, “Christ Crucified Between Two Thieves: ‘The Three Crosses’, B78ii, RVR122.





Point 1:	{262, 627}
Point 2:	{217, 460}
Point 3:	{224, 434}
Point 4:	{305, 450}
Point 5:	{311, 458}
Point 6:	{214, 397}
Point 7:	{236, 381}
Point 8:	{292, 381}
Point 9:	{316, 395}
Point 10:	{195, 304}
Point 11:	{266, 261}
Point 12:	{354, 200}
Point 13:	{194, 277}
Point 14:	{346, 169}
Point 15:	{193, 247}
Point 16:	{318, 159}
Point 17:	{200, 159}

Figure 11: Example: Left: Marked Strasbourg Bend Watermark and Table of Marked Point Coordinates, Right: Greyscale Inverted Watermark

- RVR122B78iiBetadpi400SnapWMImg2020-11-15-16-06-41.jpg
- RVR122B78iiBetadpi400TextWM2020-11-15-16-06-41.txt

The six files from previous use of `watermarkPointMarker` should all be placed in the same separate folder. The watermark images `41-1-32dpi600SnapWM2020-11-15-15-59-00.pdf` and `RVR122B78iiBetadpi400SnapWM2020-11-15-16-06-41.pdf` marked at the same seventeen points are shown (without their accompanying tables of point coordinates) in Figure 2.

The following sequence of operations introduces `visualizeOverlays9`, which will prompt you to select the folder containing the six output files from the application of `watermarkPointMarker` to the two images of similar watermarks of interest. The software will provide an animated overlay of the watermarks intended for use in identifying their differences or confirming their match.

#### A. OPEN PLAYER

Open free Wolfram CDF Player. Choose “Open” from the file menu and open `visualizeOverlays.cdf`.

#### B. ENABLE DYNAMICS

A window will appear with the red heading “Animate Two Watermark Images.” Click “Enable Dynamics” button in upper right. A window will appear as the top level document on your screen with the statement in its upper left corner “Please locate a folder of watermark Points.” This window will display the subfolders within the folder containing `visualizeOverlays`. Navigate to the folder containing the six files produced by applying `watermarkPointMarker` to the two watermarks for which an overly is desired and select it.

With only files from two marked watermarks in the selected folder, the “Animate Two Watermark Images” window will display these two images as shown in Figure 12.

#### C. CHANGE DIRECTORY

The special set-up of this guided tour precludes the need for use of the Change Directory button. However, if you want to create videos of marked watermarks in another folder, click the Change Directory

## Animate Two Watermark Images

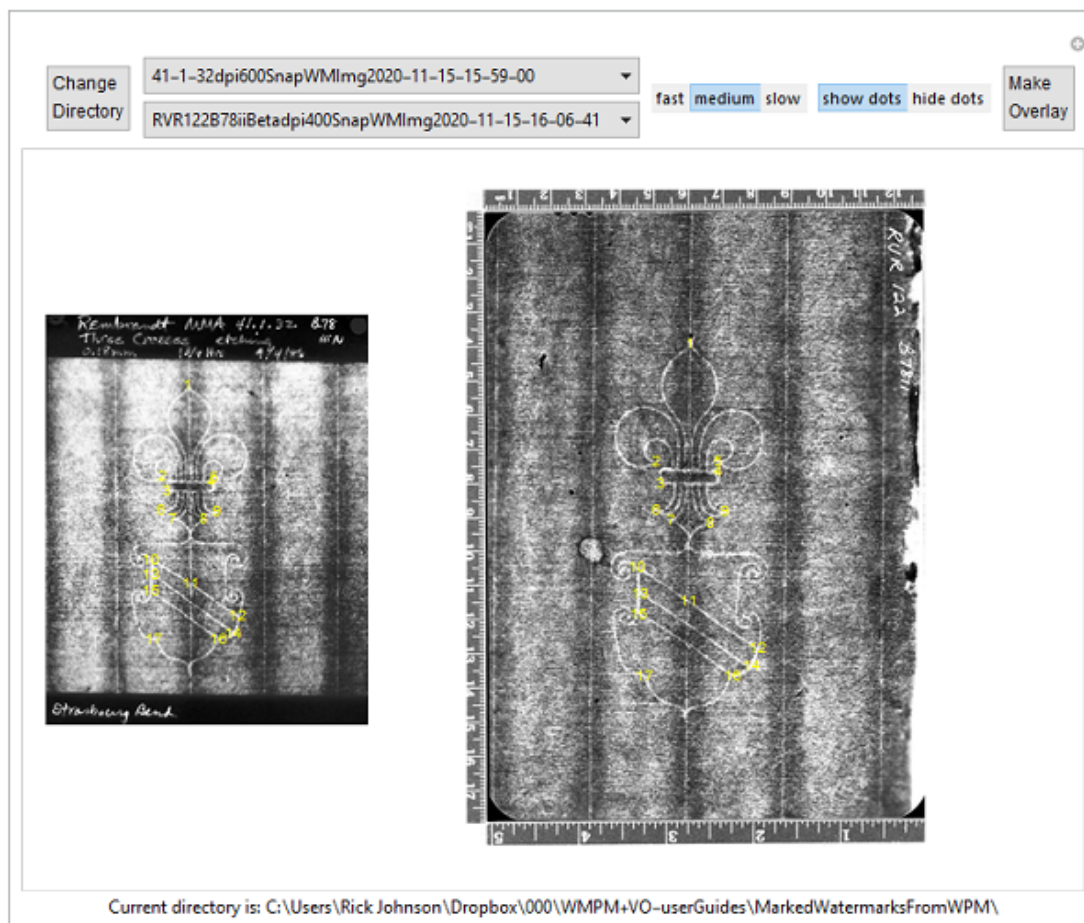


Figure 12: Animate Two Watermark Images

button, navigate to the other folder, and select it. The full name of the new folder will appear across the bottom of the window, as in Figure 12.

### D. MARKED IMAGES SELECTION

The two bars to the right of the Change Directory button in Figure 12 list two of the filenames of the SnapWMIimg files in the current folder. In this case there are only two. Moving the mouse pointer to a down arrow of either of these bars and clicking will produce a list of all available SnapWMIimg files in the current folder, from which a choice can be selected by moving the mouse pointer to hover about the choice (which will be highlighted in blue) and clicking the mouse. The watermark in the bottom bar will be the first one to appear in the video.

### E. FAST/MEDIUM/SLOW

Once the two marked watermarks have been chosen for overlay creation, two choices are to be made. One is to select from the three choices fast/medium/slow in the bar to the right of the two bars for watermark pair selection (or it may be just below the change directory button if the width of the window is constrained). The choice will determine the playback rate of frames to be recorded as the original visible watermark fades and the other watermark appears. The fast setting is the quickest to cycle one loop of the video. The slow setting produces a video that is slowest in cycling through the video loop. The medium setting is an in-between choice. The setting is chosen by hovering the mouse pointer above your choice and clicking. The selected speed will be highlighted in blue. Taking a screen shot to capture a still from the overlay video may be most easily snapped at just the right time with a slow video.

### F. SHOW DOTS / HIDE DOTS

The other choice before making an overlay video is whether or not to include the marked dots in the overlays. Selecting the choice to Hide dots in the bar to the right of the fast/medium/slow bar will display the watermarks with no obstructions and will allow closer inspection of the points at which the watermarks, if they match, should be indistinguishable. Including the marks will emphasize any errors in spot marking in identical watermarks. But, for two watermarks that are not a perfect match, including the dots can make their differences stand out more in the video.

### G. MAKE OVERLAY

To make an overlay video click the Make Overlay button. The watermark images will disappear and the statement “Now making animation ...” will appear. Once the video is completed a statement beginning with “Overlay video saved to:” will appear. Following the colon will be the address and the filename. The filename begins with “animation” followed by the names of the two marked watermarks. The filename continues with the first letters of the speed selection and the dots selection in parentheses, e.g. (ss) for slow and show dots, and ends with the file-type .gif. The address will be the folder containing `visualizeOverlays`, which is where the animation .gif file can be found.

The videos can be played by double-clicking on the file. Four example videos can be viewed for the `fh` (fast/hide dots), `sh` (slow/hide dots), `ms` (medium/show dots), and `ss` (slow/show dots) settings. Look carefully. Zooming-in to various segments of the video overlay is recommended to see subtle distinctions.<sup>6</sup> These two watermarks are not identical.

The `sh` setting was used in the linked overlay video of the two identical watermarks in the introductory summary section of this user’s guide.<sup>7</sup>

Animated .gifs, such as those generated by `visualizeOverlays`, can be played in any browser, and can be straightforwardly placed on the web just like any animated gif.

## H. CLOSING

When your use of `visualizeOverlays` is complete and you wish to close it, select “Close” from the pull down “File” menu of the Wolfram Player. In response to the question asking if you want to “Save changes to `visualizeOverlays`” click “Don’t save”.

## 4 Acknowledgements

The development of the Mathematica-based automated overlay video creation tools `watermarkPointMarker` and `visualizeOverlays` and this user’s guide was funded by Getty Foundation Research Grant ORG-201943572 “Applying Digital Image Processing Algorithms to the Study of Prints and Drawings”.

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<sup>6</sup>For Windows machine use CTRL+ for zoom-in and CTRL- for zoom-out. For Apple machine use COMMAND+ and COMMAND-.

<sup>7</sup>A more complete presentation of the context for this pair of programs and the pairs of watermarks overlaid here can be found in C. R. Johnson, Jr., W. A. Sethares, and M. H. Ellis, “Automated Overlay Videos for Accurate Watermark Identification and Moldmate Matching” submitted to the *Journal of Historians of Netherlandish Art*, January 2021.