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A POWERFUL TOOL FOR PAPER STUDIES: THE *COMPUTATIONAL CODING* OF WATERMARKED PAPERS IN LEONARDO'S CODEX LEICESTER AND CODEX ARUNDEL

Margaret Holben Ellis, William A. Sethares, and C. Richard Johnson, Jr.

Selected watermarked leaves found in the Codex Leicester (Bill and Melinda Gates Collection) and the Codex Arundel (MS 263, British Library) by Leonardo da Vinci (1452-1519) were used as case studies to evaluate the usefulness of a powerful computational tool for paper studies. The results prove that computational "coding" can contribute significantly to art historical and codicological paper research in general, and to the body of knowledge about Leonardo's papers specifically.[1]

What is Computational Coding?

Computational coding is the process whereby a researcher or "coder" examines and transforms a digital image of the watermark and chain lines present in a sheet of handmade paper into a numerical code that can be compared to codes derived from other papers.[2] A descriptive overview of the coding process is provided in Appendix II.

For watermarks, a unique set of points is designated for each watermark type. The ratios of the lengths of the lines connecting these points, together with their angles, generate a numerical code. For chain lines, the code consists of the ratios of the spaces between the chain lines. These codes are ideal for distinguishing minor variations in seemingly identical papers and can be used to confirm that the two papers are (or are not) made from the same mould.

Computational coding is straightforward and uses easily available digital photographs of the recto and verso and a transmitted light digital photograph of the paper in question.[3] The procedure involves:

- enhancement of the paper's internal structure (watermarks, chain line intervals, laid line densities) by the virtual removal of surface writing and drawing,[4]
- measurement of unique watermark features and chain line intervals and the generation of the codes,[5]
- comparison and matching of these codes to identify paper mouldmates and their probable twins.[6]

What are Paper Mouldmates and Twins?

In order to appreciate the significance of mouldmates and twins for paper studies, it is necessary to understand how they were produced. Mouldmates reflect the industrial process of papermaking in Leonardo's time, that is, one sheet was produced at a time by dipping a rectangular mould into a vat of macerated linen and/or hemp pulp mixed with water. The mould consisted of a porous screen fabricated from fine, densely spaced, horizontal rows of laid wires held in position by widely spaced, vertical chain wires. A removable wooden frame called a deckle surrounded it. When the mould was plunged into the suspended paper pulp and lifted up, the chain and laid wires acted as a sieve. The pulp was retained in thinner and thicker accumulations as the water drained away due to the interference of the laid wires and chain wires. The resulting grid-like configuration can be seen easily when the sheet is placed over a light source. The watermark, formed by stitching a thin wire bent into a simple shape onto the wires of the mould, likewise impeded the rate and quantity of pulp as it drained away, leaving behind a characteristic, translucent impression. After draining, the wet sheet of evenly distributed pulp was transferred from the mould onto a stack of freshly

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formed sheets, each made from one of two alternating moulds.

Using a pair of moulds was the most efficient means of producing a steady output of individual sheets of paper (see Figure 1, below). The vatman submerged Mould A (circled in red) into the tub, scooped up some paper pulp, and passed the mould to his helper (the coucher). Having done so, he picked up a second mould, Mould B (circled in blue), dipped it into the tub, and passed it back to his helper, who had already emptied and returned Mould A, thus creating a continuous cycle. The second mould was watermarked in the same manner as the first.[7] The paper made from this second mould, having the same type of watermark, although not identical, is called a twin. For example, if Mould A had an eagle, Mould B would as well; however, because the watermark forms were hand-fashioned, the eagles would not be identical copies. The papers were processed sequentially – stacked and pressed, then air-dried, sized, finished, and packaged. Sequence and output varied roughly according to daily or weekly production runs, and the capacity of the drying loft. Papers made from the one mould and its second, twin mould were randomly interspersed, but consistently present, in individual runs of paper production.[8] It is highly likely that Leonardo's ream or *risma* of 500 sheets, or *quaderno* of 25 sheets, contained mouldmates and twins.

Being entirely made by hand, every mould was unique, down to the uniformity of its hand-drawn wires. While at first two moulds might appear identical, subtle differences existed in the details of the watermarks, the intervals between chain wires, and the densities of the laid wires (lines per centimetre, see Figure 2). The eagle watermark might look the same from one mould to the next; however, its talons could be angled differently or its crown askew. Every sheet of paper formed from a given mould will replicate that mould's particular physical features.[9]

The Significance of Mouldmates

The designation of two sheets of paper as mouldmates means that they share a place of origin and a narrow period of manufacture, ranging from the same production run, i.e., days or weeks, to the lifespan of that particular papermaking mould. The lifespan of a mould is estimated to be anywhere from nine months to two years for a popular format produced by an active mill.11 The appearance in a manuscript of a string of mouldmates, along with probable twins, which are themselves mouldmates, implies that the papers were produced in one run and packaged together.12 A circumstantial accumulation of just identical mouldmates or their probable twins would be highly unlikely. Furthermore, the selection and arrangement of the papers prior to use could not have been an intentional act since their slight physical variations were barely visible to the naked eye.

The Codex Arundel and the Codex Leicester as Case Studies

The Codex Leicester and the Codex Arundel were excellent candidates for computational coding for three reasons:

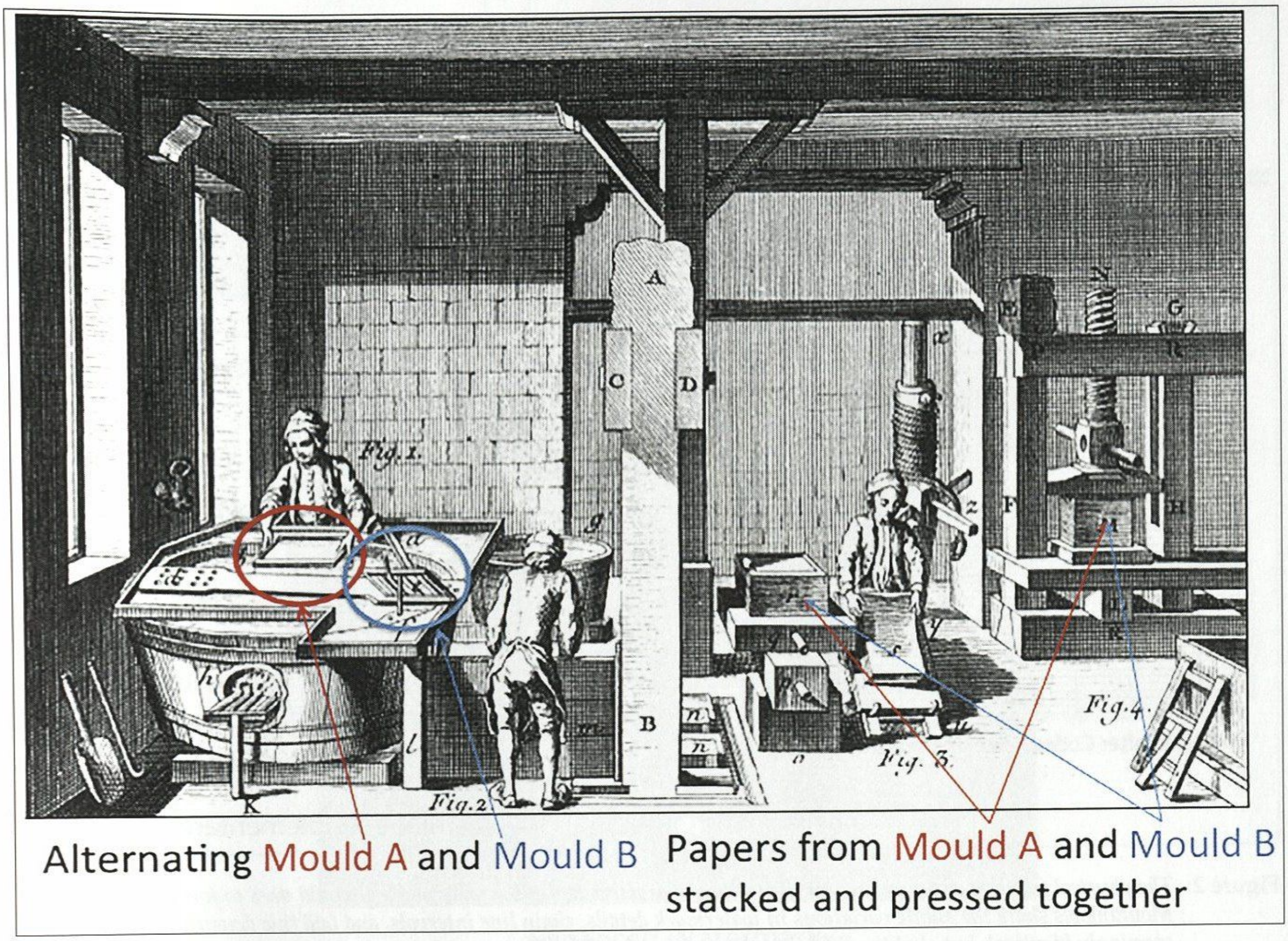

Figure 1: *In order to ensure the continuous production of individual sheets of paper, a cycle of two moulds was used by the vatman. Papers from the same mould (mouldmates) and those from the alternate mould (twins) were processed in tandem throughout each manufacturing phase. The probability of finding mouldmates and twins in the final ream of paper is high.* (Source: Diderot and d'Alembert, *Encyclopédie, ou Dictionnaire Raisonné des Sciences, des Arts et des Métiers*, Plate X, 1751-1777)

1. Digital photographs of the recto and verso[13] and transmitted light digital photographs[14] of their entire sheets were already available and accessible for almost every watermarked folio in both codices.[15]
2. Scholars have concluded that portions of the Codex Arundel and the entirety of the Codex Leicester were created during the time period of 1506 to 1518.[16] Although the proposed dates for individual sections of each codex vary slightly, given the intersection of dates, it seemed reasonable to conjecture that some of their papers were contemporaneous and, possibly, purchased in batches from a local paper purveyor. This is bolstered by the fact that the two codices share 39 examples of three watermark types: cardinal's hat, flower/tulip, and eagle.
3. The papers of the first fourteen bi-folios and half sheets of the Codex Arundel were particularly intriguing, due to their eight cardinal's hat watermarks. Because it has been suggested that Leonardo intentionally compiled this gathering as a discrete "notebook," it presented a promising source for mouldmates and twins.[17]

The Enhancement or "De-noising" of Watermarks and Chain Lines

Because dense writing and drawing on both sides characterise the folios of the Codex Leicester and the Codex Arundel, it is difficult, if not impossible, to record and decipher their watermarks and chain line intervals, much less accurately measure, compare, and match them. For this reason, the mouldmates, probable twins, and cross-codex mouldmates listed below were derived from enhanced or "de-noised" watermarks and chain lines using a process that makes the paper's internal structure more visible by virtually removing the obscuring marks on its surface.

The interference of surface marks can be suppressed by computationally processing three images: a visible light digital photograph of the recto surface of the paper over the area of the watermark; a visible light digital photograph of the verso surface over the area of the watermark; and a transmitted light digital photograph of the same area. The key idea is to "subtract out" optimally weighted versions of the two visible light images from the transmitted light image, leaving behind only the sub-surface features (watermark and chain lines) of the paper. Figure 3 shows the sequence of subtracting two visible light photographs of the recto and the verso of

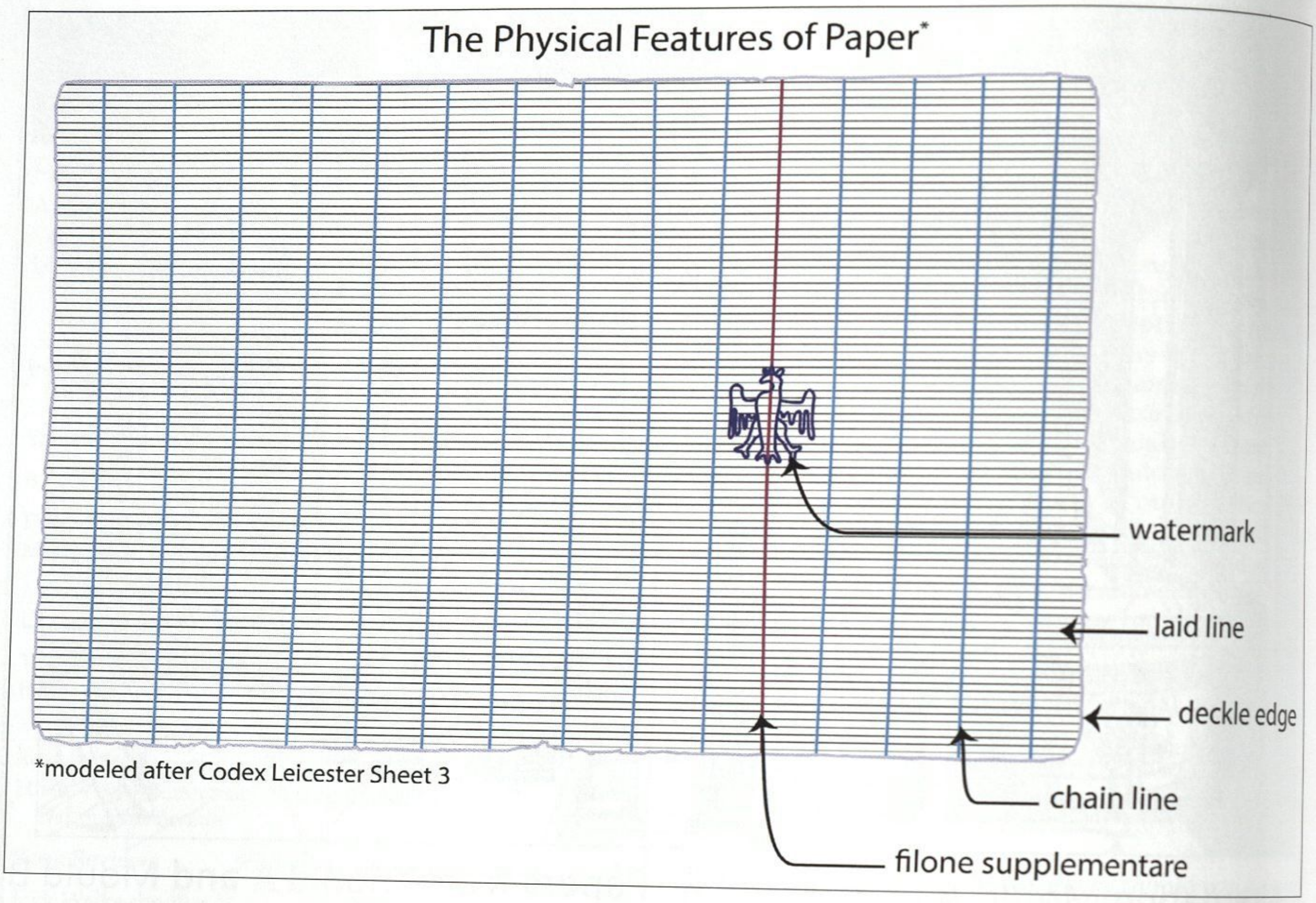

Figure 2: *The physical features of a paper reflect the unique characteristics of the papermaking mould used to form it. Mouldmates share the subtle variations in watermark details, chain line intervals, and laid line densities. Twins have seemingly identical, but slightly different, physical features. The filone supplementare is characteristic of 15th and 16th century papers produced in the vicinity of Fabriano.*[10] (Drawing by A. Slawik)

Arundel MS 263 ff. 3-12 from a transmitted light image of the same sheet. This results in an enhanced – or "de-noised" – image of the cardinal's hat watermark and of the surrounding chain lines.[18]

Mouldmate Matches in the Codex Leicester and the Codex Arundel

Computational enhancement and coding of every full or partially watermarked paper in the Codex Arundel[19] (MS 263, British Library) and the Codex Leicester[20] (Bill and Melinda Gates Collection) has indicated that nearly half (40) of the 87 sheets are mouldmates, that is, matching papers formed from one of ten different papermaking moulds.[21] Furthermore, nine of the papers appear to be cross-codex mouldmates, that is, identical sheets formed from only two papermaking moulds are shared between the Codices. This suggests the intriguing possibility that Leonardo was using the same or closely dated paper source for both manuscripts.

The Codex Leicester

The Codex Leicester, dating from ca. 1506-8 and 1510-12,[22] is a bi-folio comprised of eighteen disbound and untrimmed *rezzuta* [23] format sheets.[24] Four watermark types, listed below in order of frequency, are represented:

1. cardinal's hat,
2. eagle,
3. flower/tulip,
4. fleur-de-lis.

All the Leicester sheets were de-noised and then the watermarks and chain line intervals measured to acquire their computational codes. This showed the presence of a number of mouldmates and probable twins. As would be expected if different batches of paper from different places at different times were procured and used, strings of mouldmates and twins having cardinal's hat, flower/tulip, and eagle watermarks occur throughout the manuscript. These sequences are obvious in the collation chart of the Codex Leicester, which graphically illustrates by colour and solid and dashed lines the arrangement of mouldmates and their probable twins (see Figure 4).[25] Note that the Codex's current foliation, which is based upon records of an early 16th century binding, is depicted.[26, 27] While the traditionally accepted order of the folios cannot be confirmed as having been determined by Leonardo, scholars generally agree that Leonardo intentionally gathered together two sets of notes after they were written.[28]

From a codicological point of view, the distribution of mouldmates and twins throughout a manuscript can reflect how different paper supplies were obtained and consumed. Based solely on this assumption, the current order of the Codex Leicester seems logical. This does not, however, preclude the deliberate rearrangement of the

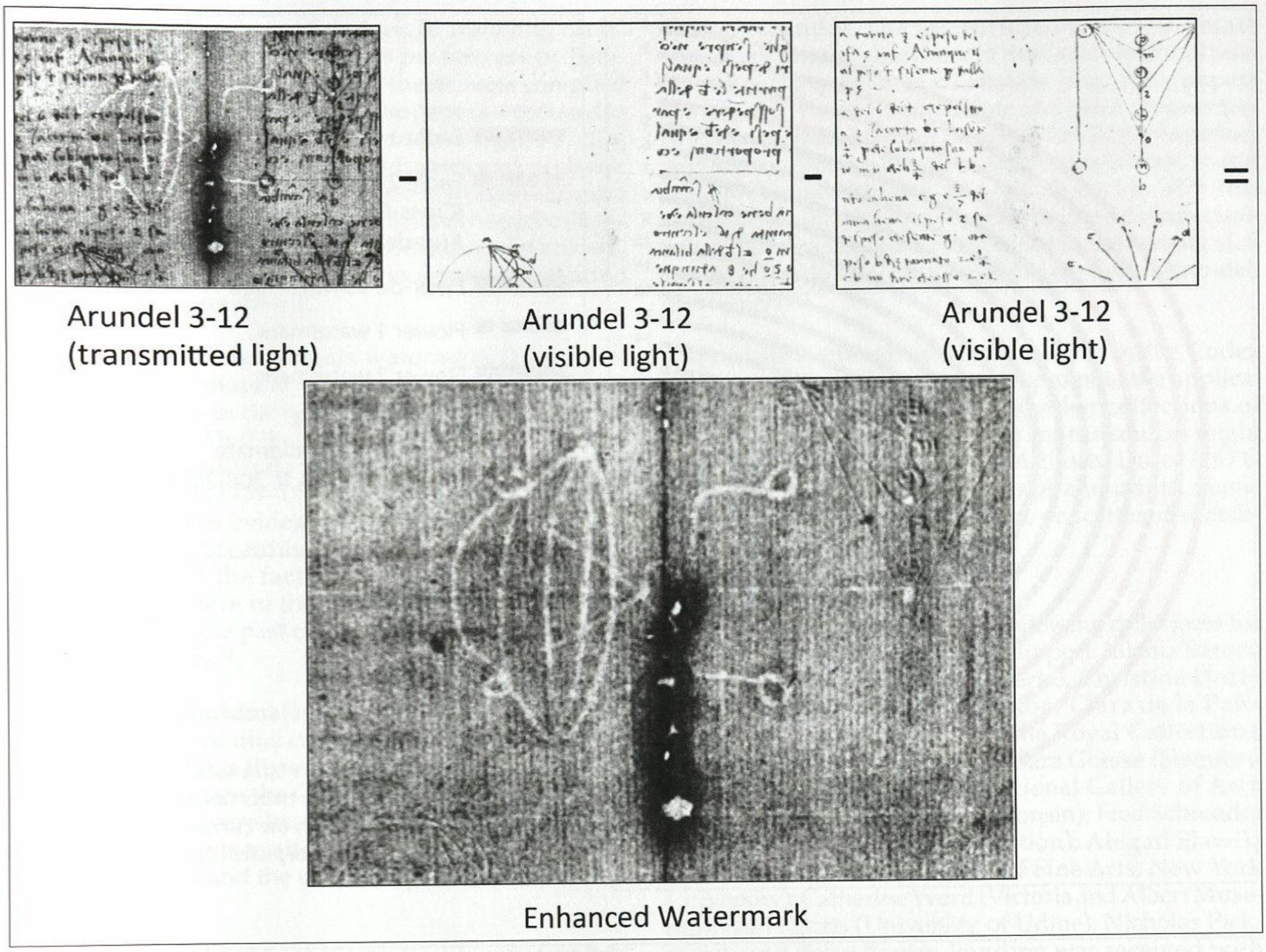

Figure 3: *Watermarks and chain line intervals are enhanced by subtracting optimally weighted visible images of the recto and verso of Arundel MS 263, ff. 3-12 (top centre and top right) from a transmitted light image of the same area (top left). The resulting image (bottom centre) has been "de-noised" of distracting surface marks.* (© British Library Board: Arundel 263 ff 3-12; processed image: William A. Sethares, Ruixue Lian)

two sets of writings at the time they were combined by Leonardo, a possibility that has intrigued Leonardo scholars for some time. Most recently, Domenico Laurenza has postulated that the Codex Leicester is made up of inner and outer sets, which "were produced relatively independently at different times, with the "outer set" compiled in a more piecemeal manner, before they were brought together and edited by Leonardo as a seventy-two page *quaderno*."[29] In this scenario, the mouldmate status of the sheet in question is still informative, since it can suggest original chronologies.

It is also possible that Leonardo selected his papers randomly, saved batches of paper over periods of time, or transported them from location to location. In this scenario, mouldmate matches might offer some clues to the artist's seemingly erratic studio practices. For example, Leicester Sheet 5, having a fleur-de-lis watermark, appears to be an outlier in its current position nestled in between eagle and flower / tulip mouldmates. If contextual evidence confirms that its location is indeed correct, its unique inclusion is still notable, especially if a fleur-de-lis mouldmate is eventually located in another collection of Leonardo papers.

The Codex Arundel

The Codex Arundel, British Library Arundel MS 263, comprised of 285 disbound bi-folia and half sheets that were assembled by a collector, dates from ca. 1478 to 1518.[30, 31] Therefore, portions of the manuscript overlap in time with the Codex Leicester. Three watermark types, listed below in order of frequency, are shared with the Codex Leicester. A fourth watermark of scissors / shears has been included for coding because of its frequency and eight mouldmate matches found elsewhere in the Codex Arundel:

1. cardinal's hat,
2. flower / tulip,
3. eagle,
4. scissors / shears (not shared with the Codex Leicester).

As in the Codex Leicester, computational coding revealed mouldmates and probable twins among the cardinal's hat, flower / tulip, and scissors / shears watermark types found throughout the Codex Arundel (listed below).

Special attention was paid to the first thirty pages of the

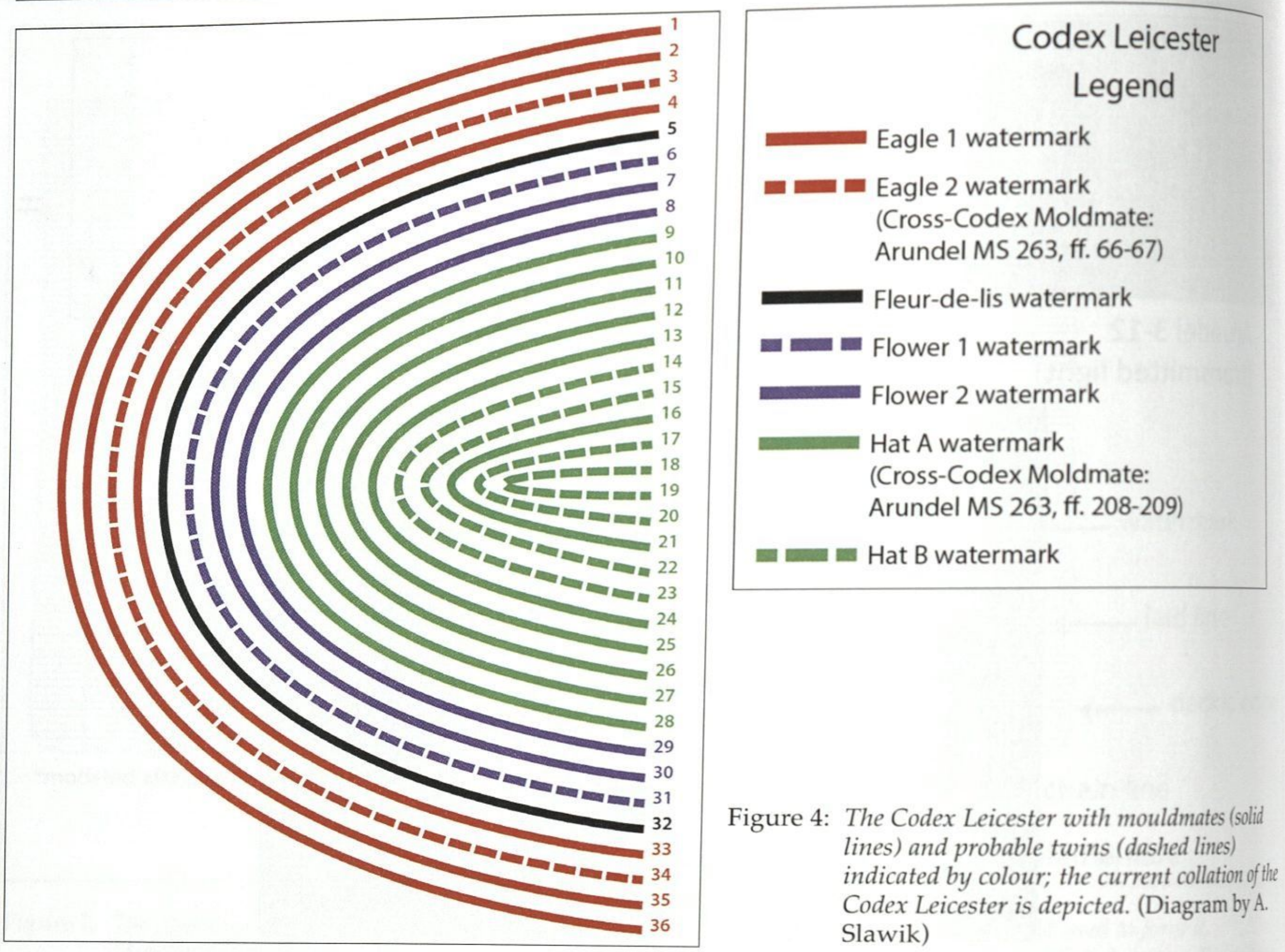

Figure 4: *The Codex Leicester with mouldmates (solid lines) and probable twins (dashed lines) indicated by colour; the current collation of the Codex Leicester is depicted.* (Diagram by A. Slawik)

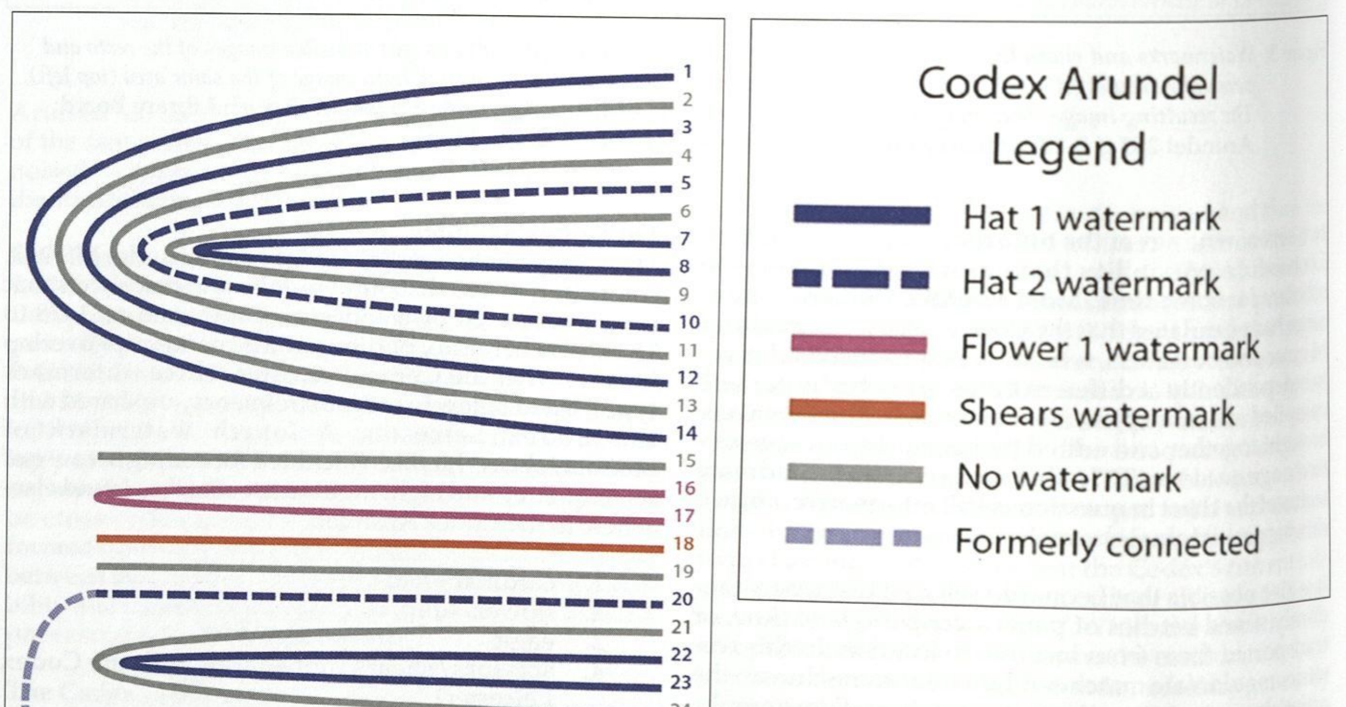

Figure 5: *Bi-folios and half sheets 1 through 30 of the Codex Arundel with mouldmates (solid lines) and probable twins (dashed lines) indicated by colour; the current collation of 1 through 30 is depicted. Note that Arundel 16r-17v (flower/tulip) and Arundel 18v (scissors/shears) have mouldmate matches elsewhere in the manuscript.* (Diagram by A. Slawik)

Codex Arundel because of their eight recurring cardinal's hat watermarks, and theories put forward by Bambach and Pedretti / Vecce that the sheets were compiled independently.[32] The codes for these papers were used to create an accurate collation diagram (see Figure 5). The sequences of cardinal's hat mouldmates and probable twins support the theories put forward by Bambach and Pedretti / Vecce and seem to confirm that the gathering is deliberate. Nine pages do not contain a watermark, which is to be expected in a quarto format constructed from a *rezzuta* format sheet.

Note that the scissors / shears watermark (Half Sheet 18v) is a mouldmate to four other Arundel bi-folios found elsewhere in the manuscript (79r-93v, 147v-148r, 271r-278v, and 273r-276v). Also, Arundel 16r-17v (flower / tulip) is a mouldmate match with two other bi-folios found elsewhere, 173v-176r and 282v-283r. Based solely on physical paper evidence, the insertion of these two sheets into a run of cardinal's hat mouldmates and probable twins, and the fact that they have mouldmate matches elsewhere in the manuscript, is notable and suggests a possible past connection between these now separated papers.[33]

Cross-codex Mouldmates

The occurrence of nine eagle and cardinal's hat cross-codex mouldmates (listed below) supports an historic connection between the Codex Arundel and the Codex Leicester. At the very least, the presence of cross-codex mouldmates points to affinities of date and place of origin of the sheets and the existence of consistent sources of paper.

No cross-codex mouldmates were found in the first thirty bi-folios and half sheets of the Codex Arundel and the Codex Leicester. While they share two common watermark types (cardinal's hat and flower / tulip), there is no physical evidence that the two manuscripts were written using the same batch or closely dated batches of paper. It seems likely that the paper came from the same local paper mills.

Computational Coding as a Tool for Paper Studies

In keeping with the current archaeological approach in the investigation of Leonardo's manuscripts, computational coding measures, compares, and matches specific physical properties of his papers.[34] While the results can illuminate fascinating patterns of mouldmates and probable twins that point to chronology and original collation, coding cannot substitute for extensive palaeographic observations, conceptual analysis of Leonardo's writings, or an in-depth knowledge of the history of their creation, agglomeration, and dispersal. Watermarks, in and of themselves, as Bambach points out, "do not provide absolute data, and must be very carefully contextualised with historical and stylistic evidence."[35] In the case of the Codex Leicester and the Codex Arundel, the coding of 47 selected watermarked sheets confirmed the presence of forty paper mouldmates and probable twins, with nine cross-codex mouldmates. Intended to be in service to scholars, these physical features support the historical connection between the two codices, the segmented, but intentionally combined, contents of the Codex Leicester, and the deliberate compilation of the first thirty bi-folios and half sheets in the Codex Arundel. The identification of mouldmate sequences located elsewhere in the Codex Arundel raises intriguing questions of collation since these papers are related by virtue of their date and place of manufacture. Mouldmate matches have been made for two other watermark types, bull's head and hunter's horn, found beyond Arundel bi-folios and half sheets 1 to 30.[36] This underscores that, besides contributing to existing scholarly propositions, coding can point to potentially rich research sites in the Codex Leicester, the Codex Arundel, and beyond.

The results of the case studies provided by the Codex Leicester and the Codex Arundel promote the application of computational coding to other collections of closely related papers. Potentially fruitful sources might include the drawing papers of Albrecht Dürer (1471-1528), large scale multi-sheet maps and prints, music scores, and disbound, reordered, or scattered sketchbooks and manuscripts.[37]

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NOTE: Caretakers of Leonardo's papers are invited to submit to the corresponding author images of the four watermark types found in Appendix I for coding.[38]

Appendix I: Paper Mouldmates and Probable Twins in the Codex Leicester and Codex Arundel

Three watermarks types are shared between the two codices: cardinal's hat, flower / tulip, and eagle. A fourth watermark type, scissors / shears, is not shared between codices, but is found in the first thirty pages of the Codex Arundel (Half Sheet 18v, see Figure 4). Because mouldmates to Half Sheet 18v are found elsewhere in the Codex Arundel, their images have been included below as an example of the intriguing connections calling for investigation by art historians and codicologists.

Listed below are the cardinal's hat, flower / tulip, eagle, and scissor / shears mouldmates and their probable twins that have been identified using computational coding. A computationally enhanced (de-noised) image of each watermark variation has been reproduced along with a simple line drawing, to be used only for rapid classification. Line drawings alone are not adequate for mouldmate matching.

CARDINAL'S HAT

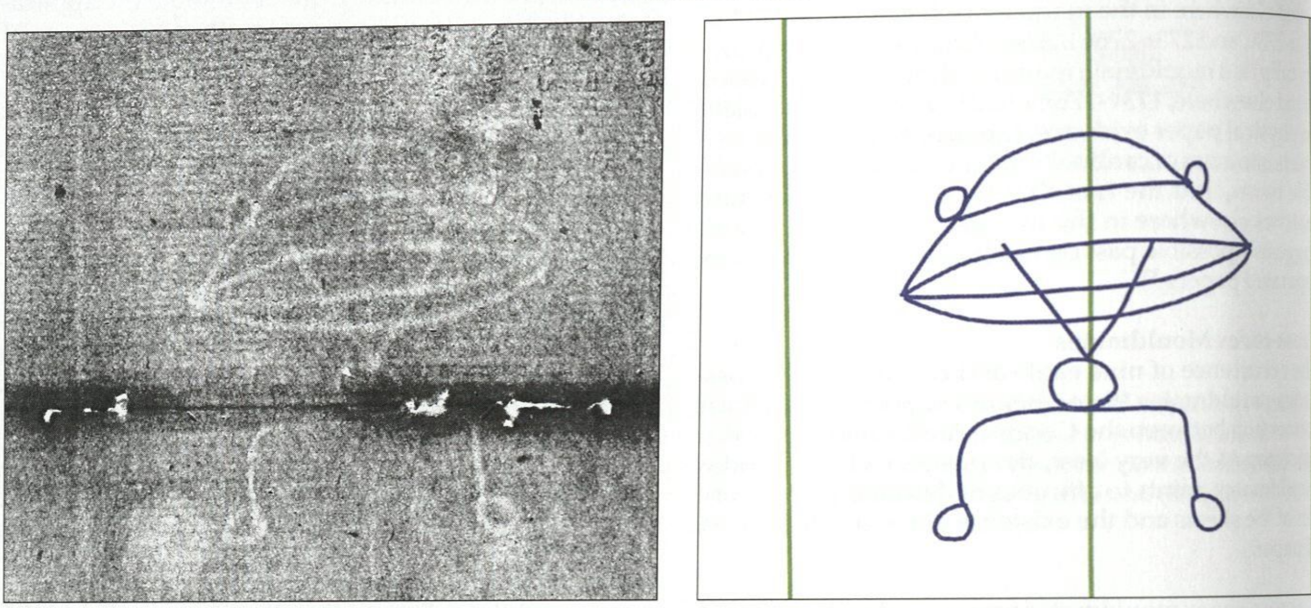

Figure 6: *HAT 1 from Arundel 22r-23v: de-noised image, left; line drawing, right.*

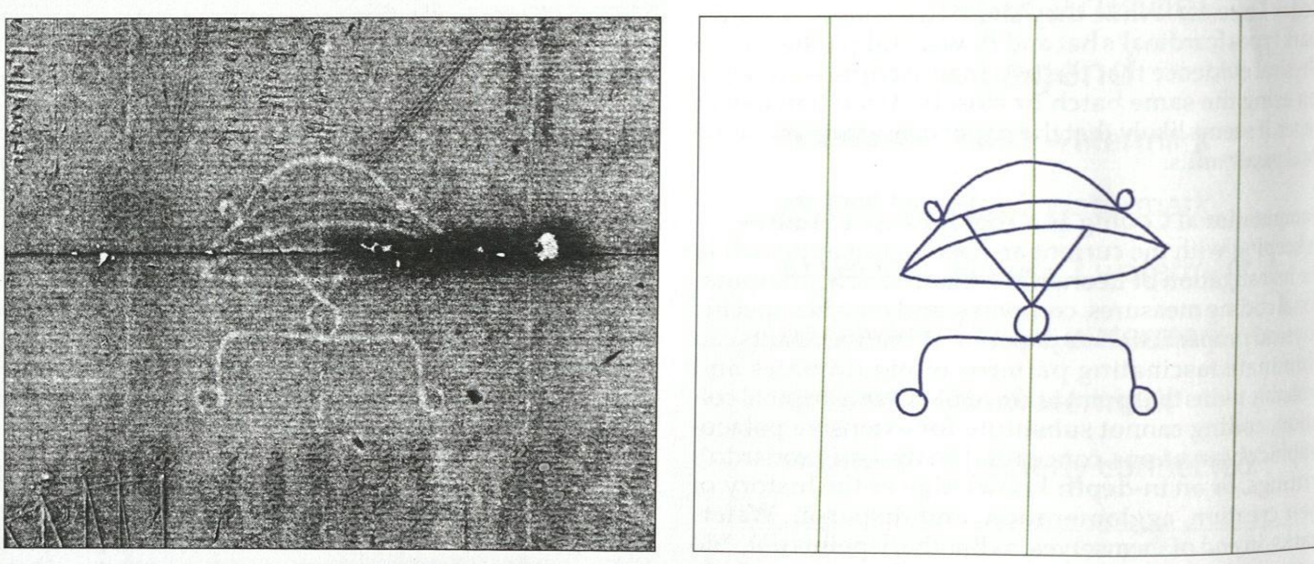

Figure 7: *HAT 2 from Arundel 5r-10v: de-noised image, left; line drawing, right.*

HAT 1 Mouldmates	HAT 2 Mouldmates (Twins)[39]	HAT A Cross-codex Mouldmate[40]	Other HAT watermarks
1v-14r (115)[41]	5r-10v (119)	208r-209v	207v (43)* [42]
3r-12v (117)	20v-28r (132)		152v (13)
7r-8v (121)			
2r-23v (128)			
26v (130)			
30v (134)			

Table 1: *Details of HAT watermarks in the Arundel Codex.*[43]

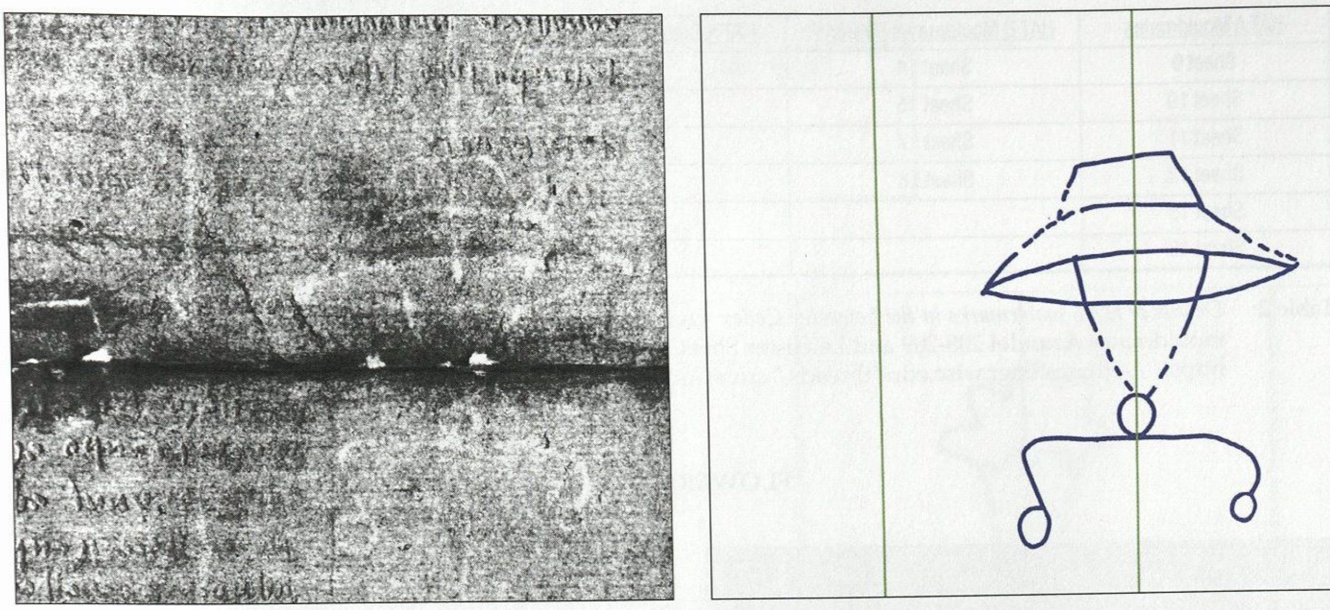

Figure 8: *HAT A from Arundel 208r-209v; de-noised image, left; line drawing, right.*

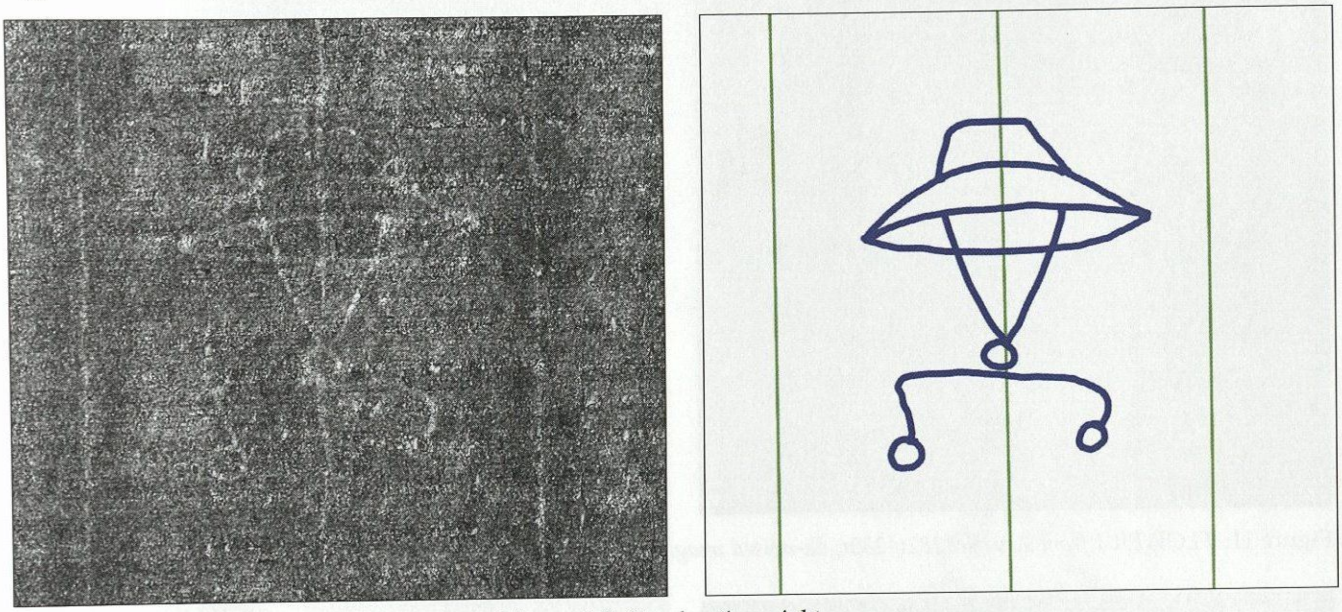

Figure 9: *HAT A from Leicester 12: de-noised image, left; line drawing, right.*

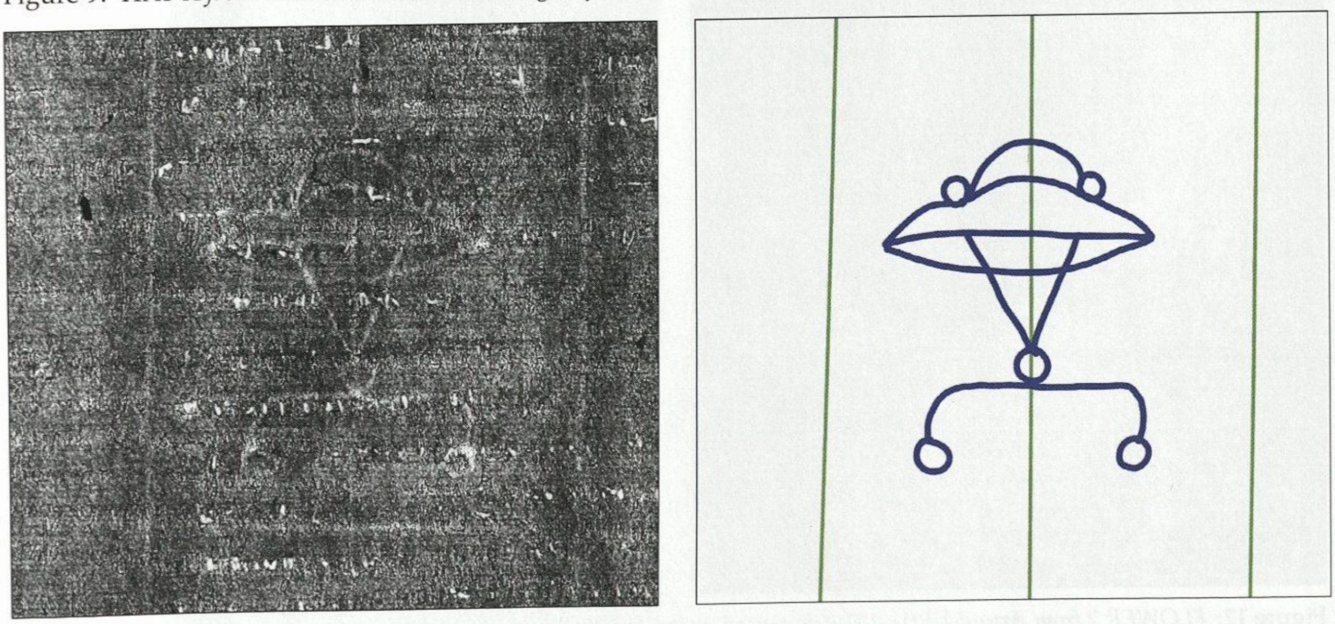

Figure 10: *HAT B from Leicester 18: de-noised image, left; line drawing, right.*

HAT A Mouldmates	HAT B Mouldmates (Twins)[44]	HAT A Cross-codex Mouldmates
Sheet 9	Sheet 14	Sheet 9
Sheet 10	Sheet 15	Sheet 10
Sheet 11	Sheet 17	Sheet 11
Sheet 12	Sheet 18	Sheet 12
Sheet 13		Sheet 13
Sheet 16		Sheet 16

Table 2: *Details of HAT watermarks in the Leicester Codex*. (A looping overlay video of the HAT A cross-codex mouldmates Arundel 208-209 and Leicester Sheet 12 can be viewed at: https://sethares.engr.wisc.edu/threads/ crossCodexAnimation2.gif)

FLOWER/TULIP

Figure 11: *FLOWER 1 from Arundel 282v-283r; de-noised image, left; line drawing, right.*

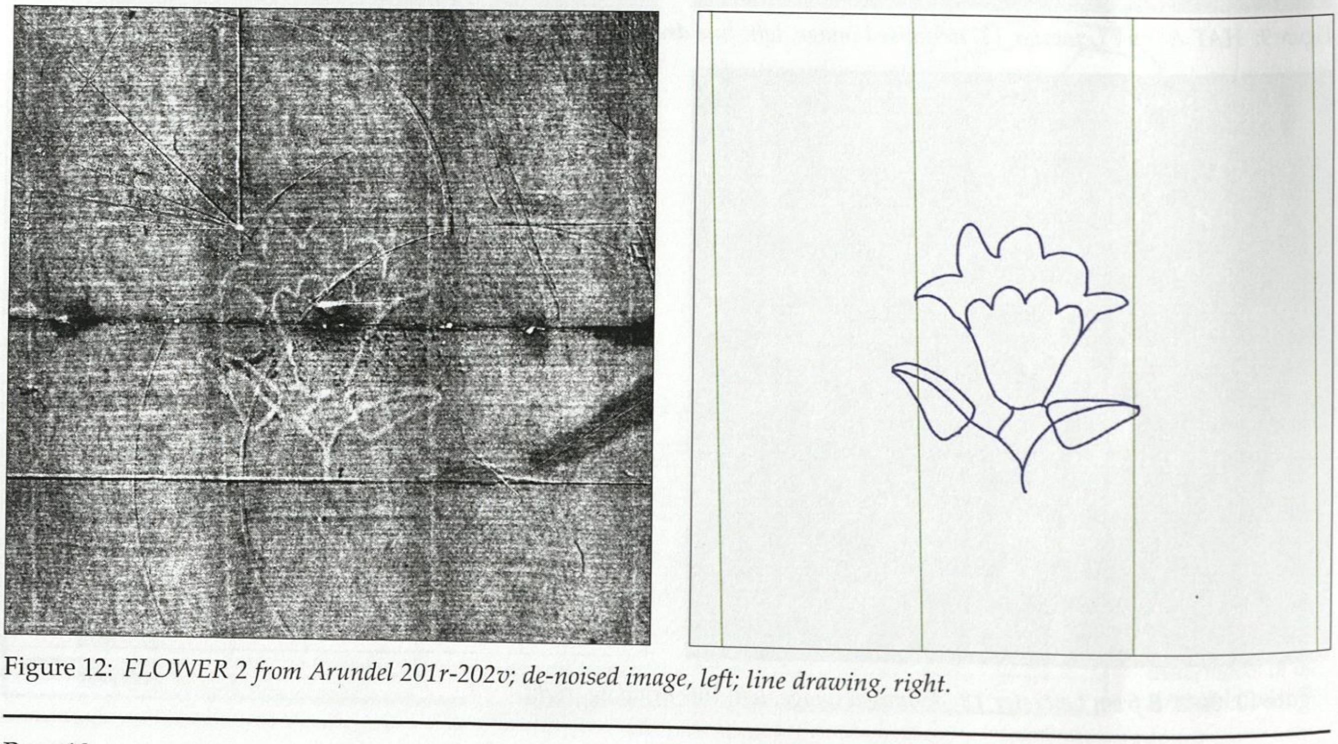

Figure 12: *FLOWER 2 from Arundel 201r-202v; de-noised image, left; line drawing, right.*

FLOWER 1 Mouldmates	FLOWER 2 Mouldmates[45]	Other FLOWER watermark
16r-17v (123)	104v-107r (105)	178r-179v (92)
173v-176r (75)	201r-202v (45)	
282v-283r (84)	212v-213r (37)	
	237v-242r (73)	

Table 3: *Details of FLOWER watermarks in the Arundel Codex.*

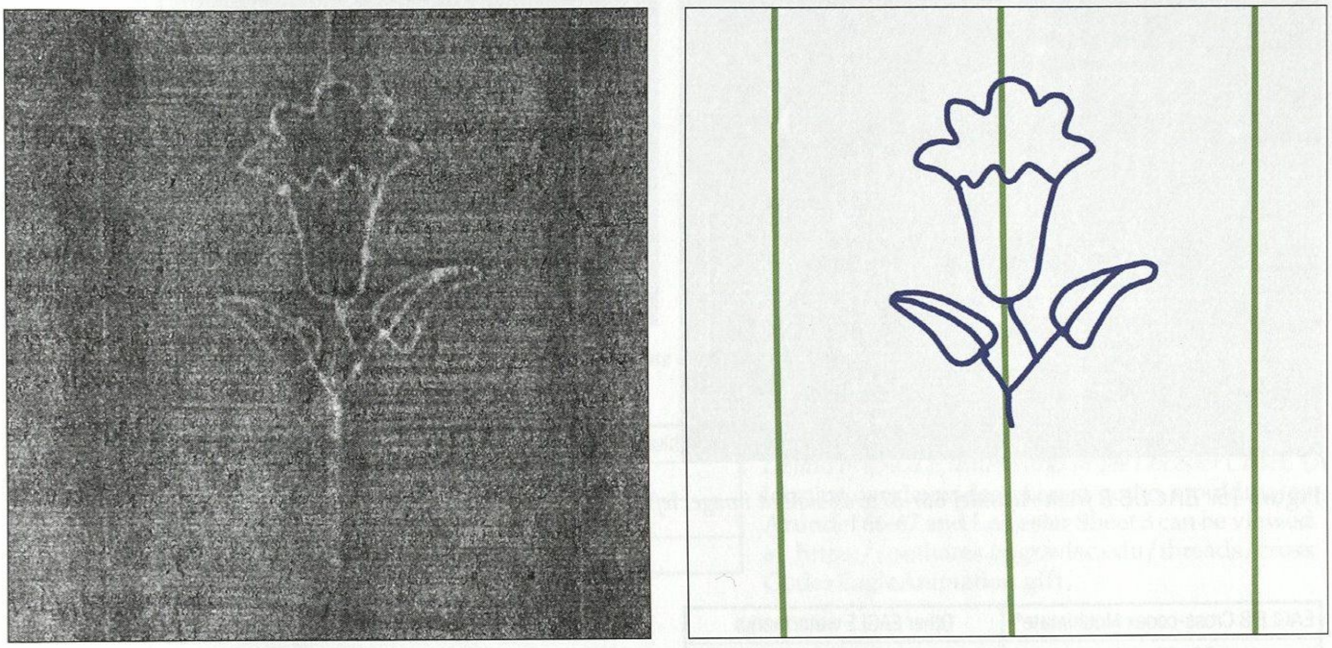

Figure 13: *FLOWER A from Leicester 7; de-noised image, left; line drawing, right.*

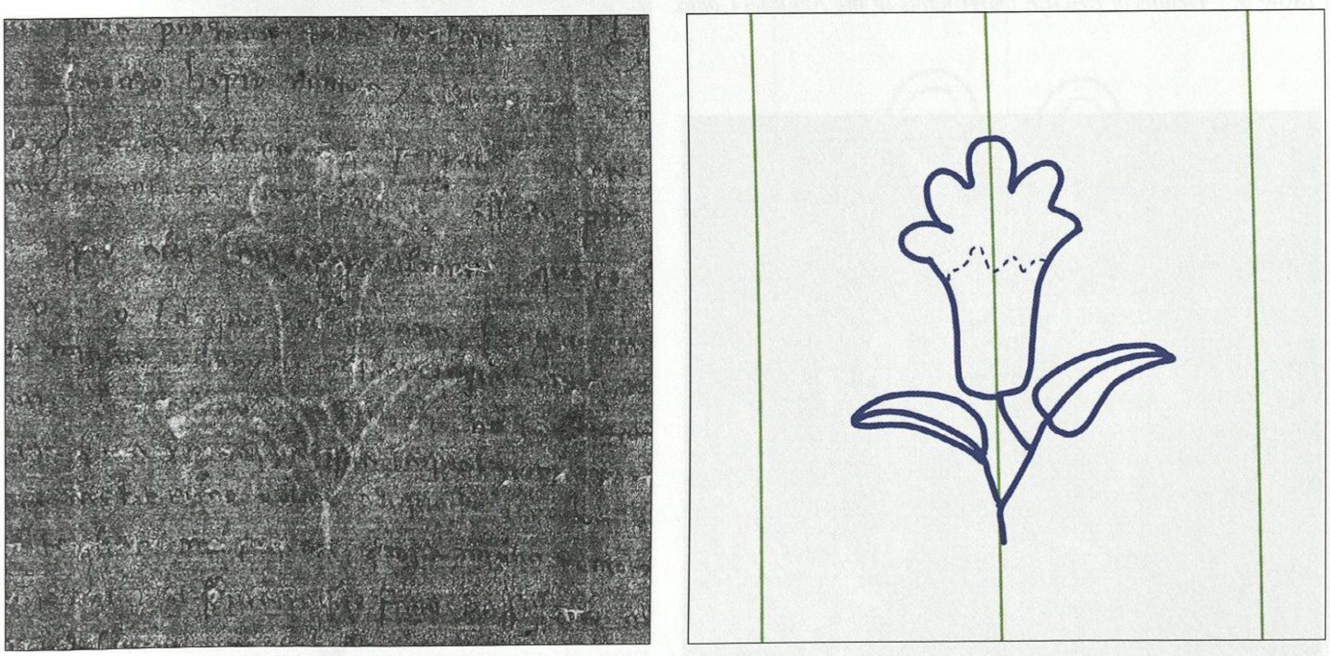

Figure 14: *FLOWER B from Leicester 6; de-noised image, left; line drawing, right.*

FLOWER A Mouldmates	FLOWER B Twin[46]
Sheet 7	Sheet 6
Sheet 8	

Table 4: *Details of FLOWER watermarks in the Leicester Codex.*

EAGLE

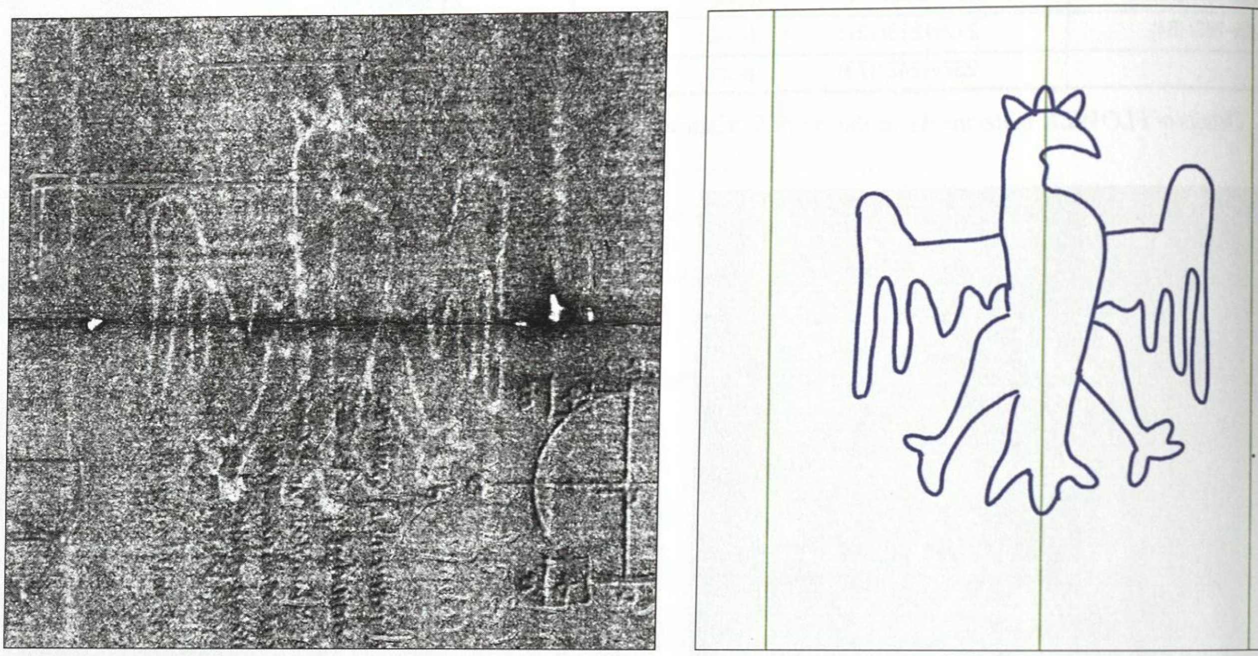

Figure 15: *EAGLE B from Arundel 66r-67v; de-noised image, left; line drawing, right.*

EAGLE B Cross-codex Mouldmate[47]	Other EAGLE watermarks
66r-67v (49)	219v-222r (97)*
	239r-240v (98)

Table 5: *Details of EAGLE watermarks in the Arundel Codex.*

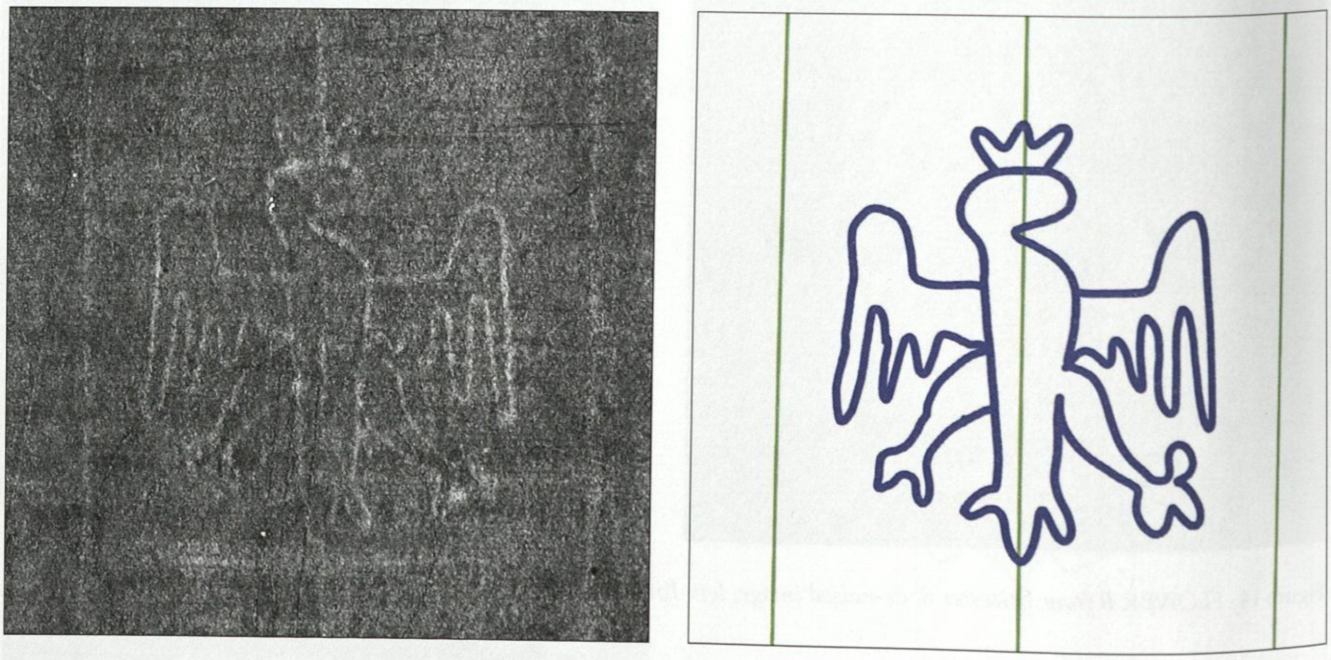

Figure 16: *EAGLE A from Leicester 1; de-noised image, left; line drawing, right.*

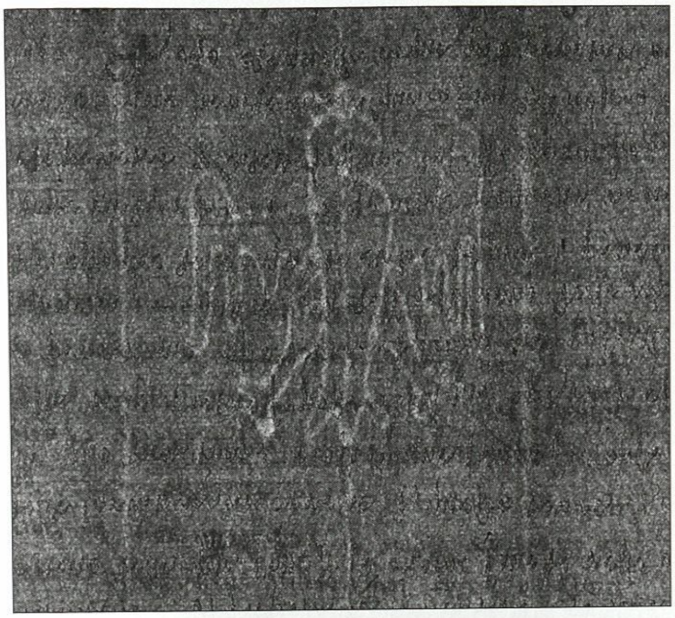
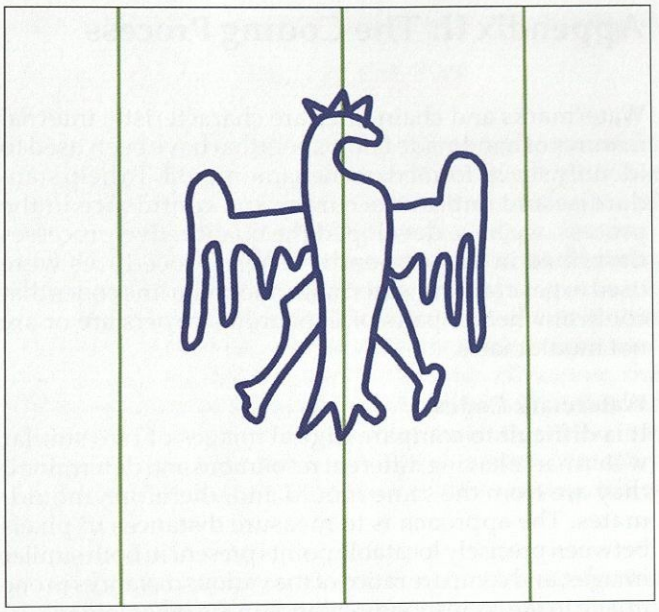

Figure 17: *EAGLE B from Leicester 3; de-noised image, left; line drawing, right.*

EAGLE A Mouldmates	EAGLE B Twin[48]	EAGLE B Cross-codex Mouldmate
Sheet 1	Sheet 3	Sheet 3
Sheet 2		
Sheet 4		

Table 6:
Details of EAGLE watermarks in the Leicester Codex. (A looping overlay video of cross-codex mouldmates Arundel 66-67 and Leicester Sheet 3 can be viewed at: https: / / sethares.engr.wisc.edu / threads / cross CodexEagleAnimation.gif)

SCISSORS/SHEARS

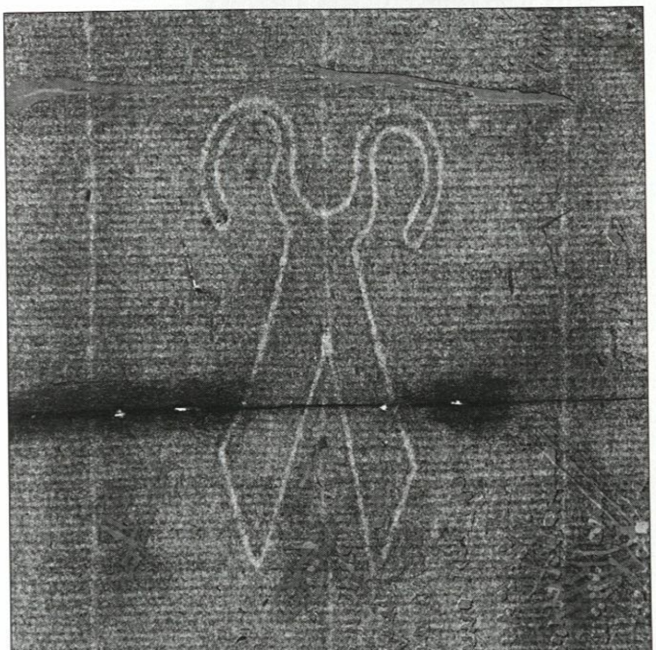
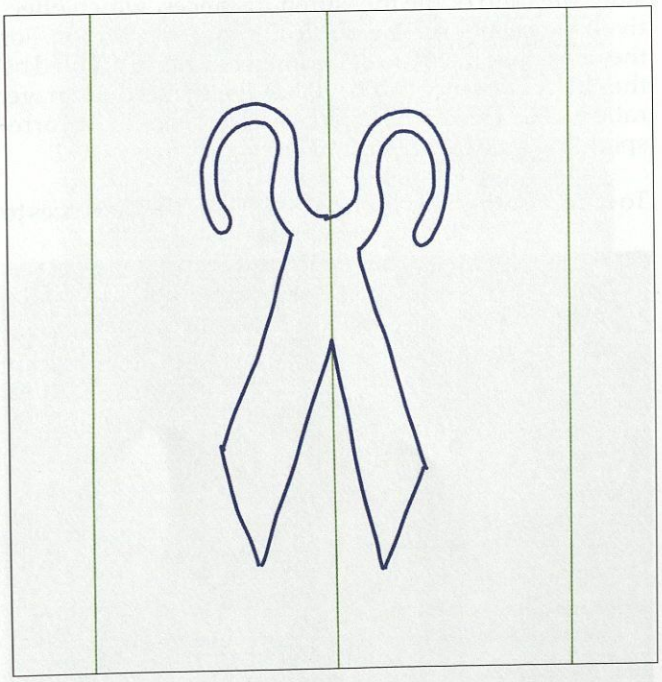

Figure 18: *SCISSORS 1 from Arundel 273r-276v; de-noised image, left; line drawing, right.*

Scissors 1 Mouldmates
18 (127)
56-93 (85)
147-148 (82)
271-278 (78)
273-276 (80)

Table 7:
Details of SCISSOR watermarks in the Arundel Codex.

Appendix II: The Coding Process

Watermarks and chain lines are characteristic internal features of handmade laid papers that have been used to identify sheets formed on the same mould. To help standardise and improve accuracy and confidence in the process, we have developed the quantitative processes described in this Appendix. These procedures were used to generate the watermark and chain line codes that confirm whether pairs of Leonardo's papers are or are not mouldmates.

Watermark Codes

It is difficult to compare digital images of two similar watermarks having different resolutions and determine if they are from the same mould and, therefore, mouldmates. The approach is to measure distances in pixels between precisely locatable points present in both similar images, and compare ratios of the various distances in one image to the corresponding ratios in the other image.

For example, consider a right triangle with the lengths of its vertical and horizontal legs, i.e., not the hypotenuse, providing the two distances measured in both images. The length of one leg in the original object is one inch. The length of the other leg is two inches. The first digital image is captured at a resolution of 100 pixels per inch. Thus, the two distances in this digital image are 100 and 200 pixels. The second digital image is captured at a resolution of 150 pixels per inch, and the two distances in this second digital image are 150 and 300 pixels. Measured in pixels, the distances do not match. Instead, consider the ratio of the measured distances, which effectively cancels the scaling due to the image resolution. For the first image, the ratio of the longer distance divided by the shorter distance is 200/100=2. For the second image, ratio is 300/150=2. They match each other and the corresponding ratio in the original object, which is 2/1=2.

To minimise the human error in marking the distances to be compared, the end points selected must be precisely locatable points in the watermark image or on its nearby chain lines. The left image in Figure 20 is the de-noised scissors watermark in Arundel 042r-043v. The right image in Figure 20 marks three precisely locatable points: (i) the lower tip of the left blade, (ii) the upper intersection of the vertical chain line and the watermark, and (iii) the lower tip of the right blade. We are interested in the ratio of the distance from (i) to (ii) and the distance from (ii) to (iii) as an indicator to be tested in the digital image of a similar watermark. If the ratios of the same two lines in two digital images are different, the two watermarks are not mouldmates. An animated example of the placement of the red diamonds in a recently discovered set of hunter's horn mouldmates (Arundel 34v-35r and 122v-125r) can be viewed at https://sethares.engr.wisc.edu/threads/hornWatermarkAnimation.gif

To be more confident in declaring two watermarks to be mouldmates, we would mark several lines in the two watermarks being compared. The seven lines chosen for an Arundel scissors watermark are shown in Figure 21. Note that lines 1 and 2 in Figure 21 are, respectively, the lines between points (i) and (ii) and between points (ii) and (iii) on the right in Figure 20. For the seven lines in Figure 21 the ratios of the lengths of line 6 divided by the lengths of lines 1 through 7 produce the Watermark Code sequence for Arundel 042r-043v:

0.63, 0.66, 0.96, 1.03, 1.23, 1.00, 1.42

The marked lines at the corresponding locations on the scissors watermarks in Arundel 147v-148r and Arundel 273r-276v are shown in Figure 22. The Watermark Code sequence for Arundel 147v-148r is

0.59, 0.60, 1.06, 0.96, 1.11, 1.00, 1.41

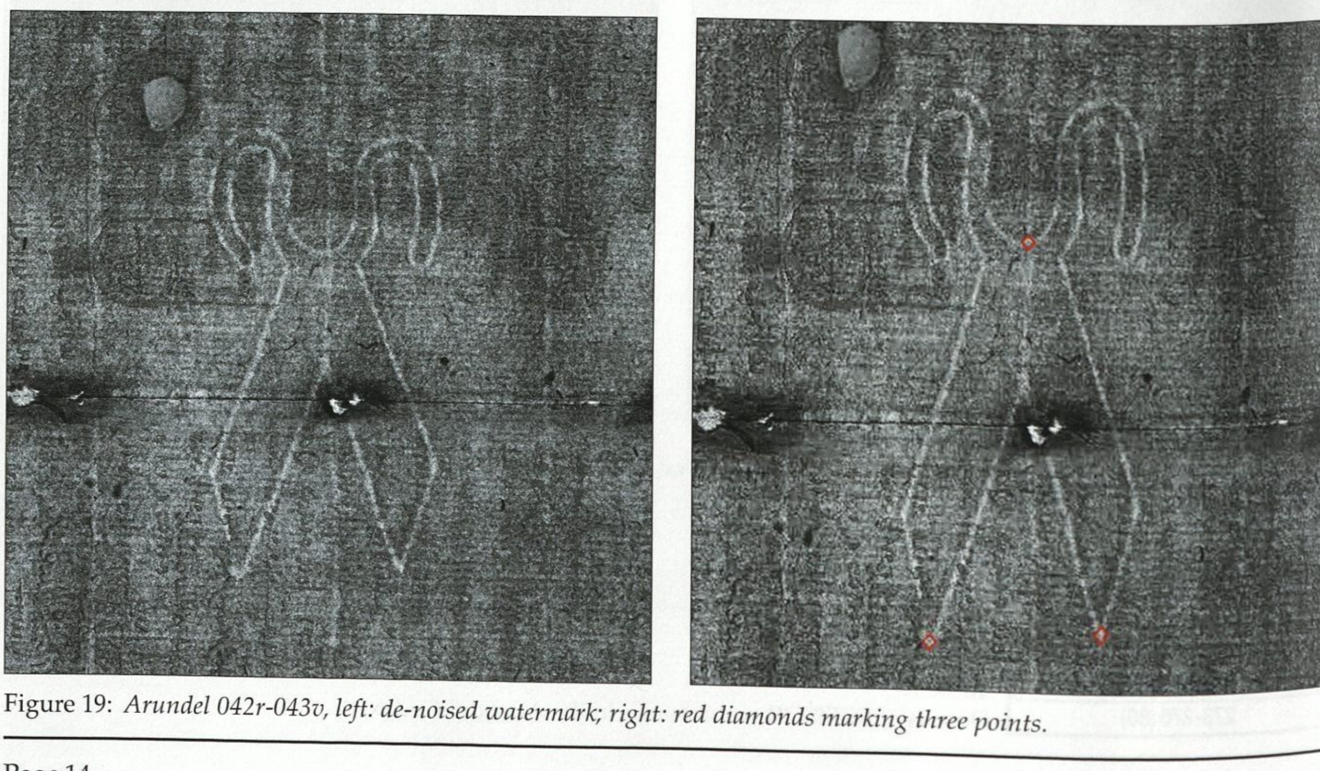

Figure 19: *Arundel 042r-043v, left: de-noised watermark; right: red diamonds marking three points.*

Figure 20: *Arundel 42r-43v: seven lines are used to compose a watermark code.*

For Arundel 273r-276v, the Watermark Code sequence is

0.61, 0.61, 1.09, 0.98, 1.11, 1.00, 1.42

Given the inaccuracies in marking the chosen points in the images, we choose to declare two Watermark Code sequences the same only if all entries are within 0.1 of their corresponding value in the other image. Furthermore, the range of values across a group of mouldmates should be 0.1 or less for all entries. The fifth value in the sequence for Arundel 42r-43v fails this requirement in comparison to the fifth values in the sequences for Arundel 147v-148r and Arundel 273r-276v. However, the Watermark Code sequences for Arundel 147v-148r and Arundel 273r-276v do satisfy the matching criterion. Therefore, Arundel 147v-148r and Arundel 273r-276v are mouldmates, while Arundel 42r-43v is from a different mould.

In order to be even more confident, we also derive Watermark Angle codes by recording the angles between pairs of the seven marked lines that share an end point. In this case, the line pairs forming angles are 1-2, 3-4, 4-5, 5-6, and 6-7. All angles in a Watermark Angle sequence must be within three degrees of the corresponding angle for two watermarks to be declared mouldmates. The results for the Watermark Angle Code sequences for Arundel 42r-43v, Arundel 147v-148r, and Arundel 273r-276v are the same as the mouldmate designations from the Watermark Codes. The difference is quite visible in Figure 22. The angle formed by lines 3 and 4 is less than 90 degrees in Arundel 42r-43v in Figure 21 while in Figure 22 showing Arundel 147v-148r and

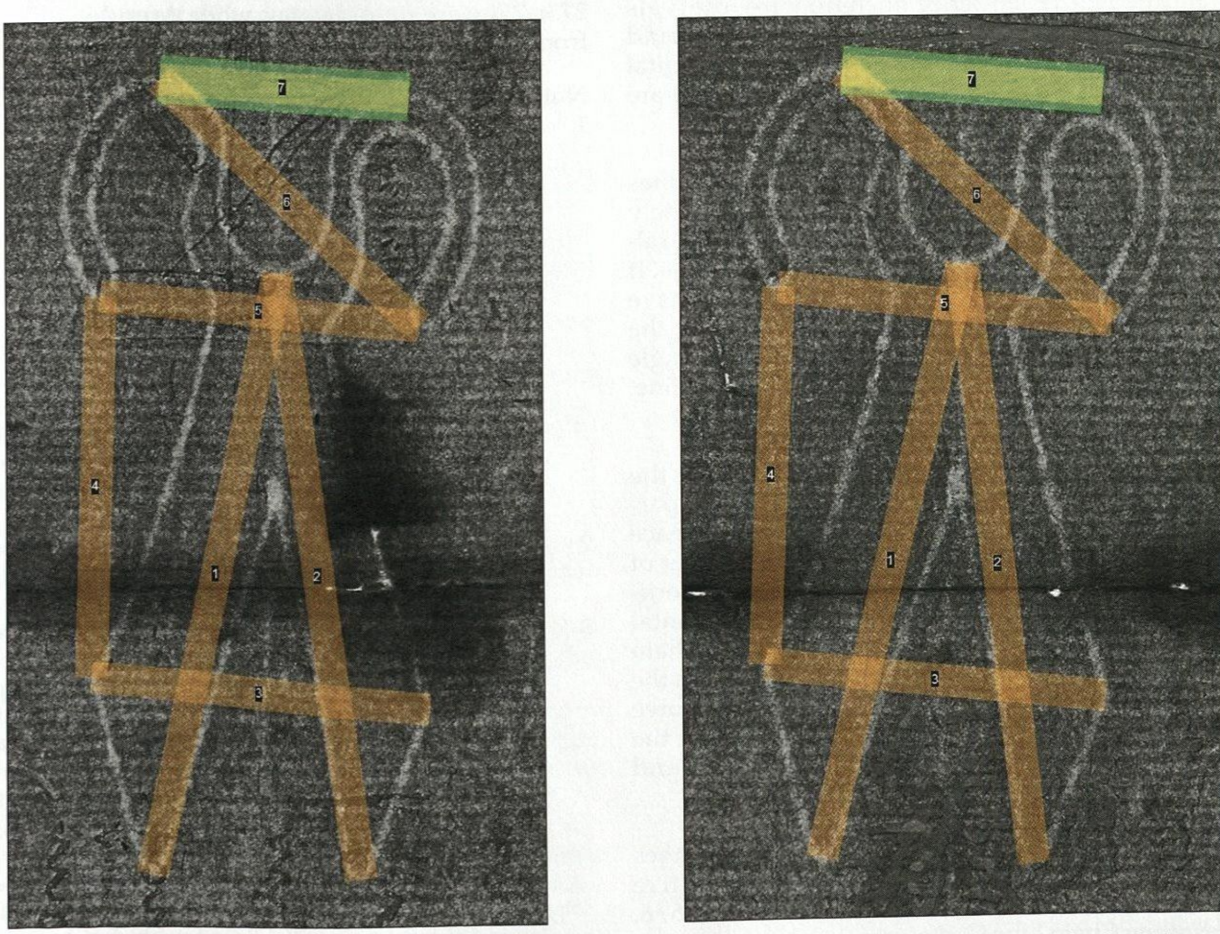

Figure 21: *Additional scissors watermarks from the Codex Arundel, left: Arundel 147v-148r; right: Arundel 273r-276v.*

Figure 22: *Marked stripes, top: Arundel 42r-43v; middle: Arundel 147v-148r; bottom: Arundel 273r-276v.*

Arundel 273r-276v it is greater than 90 degrees.

In this example, a visible watermark feature distinguishes Arundel 42r-43v from the mouldmate pair Arundel 147v-148r and Arundel 273r-276v. The gap in the left handle at the top of the scissors in Arundel 42r-43v is visibly narrower than the similar gaps in Arundel 147v-148r and Arundel 273r-276v. This provides confirmation of the validity of our procedure.

Chain Line Codes

Chain lines in the papers in question are spaced approximately one inch apart. Because papermaking moulds are handmade, the sequence of chain wire intervals across one mould varies modestly, and also from mould to mould. As with digital images of watermarks, digital images of sheets of paper revealing the chain lines are frequently collected at different resolutions.

Furthermore, the images may not have the chain lines oriented precisely vertically. Therefore, an accurately measured interval will depend on the angle of the parallel chain lines relative to the horizontal marking line. If the chain lines are perfectly vertical, and therefore have a 90 degree angle with the horizontal marking line, the distance will be smaller than for any other relative angle between the chain lines and the horizontal marking line. This is another issue that is resolved by using ratios.

A final concern that needs to be addressed is that the chain lines may not be perfectly straight or precisely parallel to each other. Consequently, the chain line space sequence for a horizontal line drawn across a sheet of paper can be slightly different if taken at different vertical coordinates. To mitigate this affect, the horizontal marking line crossing the approximately vertical chain lines should be placed at the same location in each of the sheets being compared. For example, consider the three horizontal stripes drawn across the bottom tips of the scissors in Arundel 42r-43v, Arundel 147v-148r, and Arundel 273r-276v as illustrated in Figure 23.

The Chain Line Code sequence in these three images with the six spaces numbered left to right are the five ratios of the space widths 1/2, 2/3, 3/4, 4/5, and 5/6. The resulting Chain Line Codes are:

Arundel 42r-43v	0.96, 1.16, 1.18, 0.76, 0.97
Arundel 147v-148r	1.02, 1.20, 1.00, 0.83, 0.97
Arundel 273r-276v	1.02, 1.21, 0.99, 0.82, 0.97

The blue number is the ratio spanning the watermark. This ratio must be aligned for a proper comparison of a pair of watermarks. The tolerance allowed for mouldmate declaration is the same as for the Watermark Codes, i.e., within a range of 0.1 in every entry. By this criterion, the same match assessment is reached as with the Watermark Codes. Arundel 147v-148r and Arundel 273r-276v are mouldmates, while Arundel 42r-43v is from a different mould.

Notes and References

1. The authors would like to recognise the ongoing accumulation and cross-referencing of Leonardo's watermarks undertaken by Martin Clayton (The Royal Collection), the detailed documentation of watermarks found in the Codex Arundel by Carlo Pedretti and Carlo Vecce (Carlo Pedretti and Carlo Vecce, *Il Codice Arundel 263 nella British Library: edizione in facsimile nel riordinamento cronologico dei suoi fascioli*, Florence, 1998), and the ambitious project to transcribe watermarks found in the Codex Atlanticus (Biblioteca Pinacoteca Ambrosiana) by Claudio Calì under the leadership of Pietro C. Marani and presented at *Decoding Leonardo's Codices*, hosted by the Kunsthistorisches Institut and the Museo Galileo, Florence, November 2019. A recording of the entire symposium is found at https://vimeo.com/showcase/6528343
2. While the acquisition of codes may appear complex, user-friendly tools, and tutorials for using them, will facilitate the process.
3. An image of the entire sheet of paper, not just the area of the watermark, is optimal because it includes more chain line intervals for measurement and comparison. The principal requirement for the images is legibility—if a feature, such as a watermark, can be seen clearly, the information needed to construct the computational codes can be detected and extracted. All three digital images should be taken at the same resolution, which is sufficient for digital alignment (300-600 DPI is suitable). The two surface images should be photographed under identical and even lighting. The transmitted light image should be lit from behind by bright, even illumination, which is sufficient to

reveal internal features, but not so intense as to wash out their subtler edges.

4. The software required to enhance watermarks will be part of a package, produced with the help of the Getty Foundation, to be distributed under an open-source license. A more detailed description of the process may be found in William A. Sethares, Margaret Holben Ellis, and C. Richard Johnson, Jr., "Computational Watermark Enhancement in Leonardo's *Codex Leicester*," *Journal of the American Institute for Conservation*, Vol.59, No.2, March 2020, pp.87-96, https://doi.org/10.1080/01971360.2019.1703483.
5. It is planned that the necessary programs, *watermarkMarker* and *chainLineMarker*, will be available as free and open-source software.
6. Tutorials are being developed that will allow institutions to code and detect mouldmate matches within groups of related papers in their collections.
7. The watermarks on the alternating pair of moulds were placed on the centre right half and the centre left half sides respectively (Sylvia Rodgers Albro, *Fabriano: City of Medieval and Renaissance Papermaking*, New Castle and Washington, D.C., 2016, p.69). Their original location can be determined by their coordinates as measured from the wire side of the sheet. It was not possible to differentiate the wire side from the felt side from images of the Codex Leicester and the Codex Arundel.
8. Allan Stevenson, "Watermarks are Twins," *Studies in Bibliography*, Vol.4, 1951-52, pp.87-90; Paul Needham, "The Study of Paper from an Archival Point of View," *IPH Yearbook*, 7, 1988, p.124.
9. Substantial progress has recently been made in producing laid line density maps that can be used to match papers lacking watermarks. See Sara F. Gorske, C. Richard Johnson, Jr., William A. Sethares, Margaret Holben Ellis, Paul Messier, "Mouldmate identification in pre-19th-century European paper using quantitative analysis of watermarks, chain line intervals, and laid line density," *International Journal for Digital Art History*, forthcoming.
10. See Albro, Ref 7, p.147.
11. The lifetime of a papermaking mould is dependent upon its dimensions, with popular sizes being more short-lived. *Rezzute*, large quantities of which were in demand by book publishers, were also known as *fogli comuni* (leaves of a common size). A three to four-year lifespan of moulds has been suggested, but given the rigors of their continuous and rough use, this seems unrealistic (Albro, Ref 7, p.84). Estimates for the duration of typical moulds according to Neil Harris (personal communication) and Peter Bower (personal communication). Additionally, moulds could be sold as material assets and change their place of origin. That is why the position of one watermarked sheet relative to other identical watermarked sheets within a manuscript is critical.
12. It is important to note that when a mouldmate occurs out of sequence, no assumptions can be made regarding its twin status or its time of manufacture relative to mouldmates found elsewhere in the manuscript.
13. Digitized visible images of all 285 folios are freely available at: http://www.bl.uk/manuscripts/FullDisplay.aspx?ref=Arundel_MS_263.
14. The authors are grateful to Dr. Andrea Clarke, Curator, British Library, for generously sharing these transmitted light images. In addition to viewing differences in paper translucency via transmitted light, there are other means of detecting and recording the inner structural features of paper with varying degrees of accuracy; see Margaret Holben Ellis and C. Richard Johnson Jr., "Computational Connoisseurship: Enhanced Examination Using Automated Image Analysis," *Visual Resources*, Vol.35, No.1-2, 2019, pp.125-140. Harris has written extensively about the history of reproducing watermarks and the advantages and disadvantages of each method, in "Paper and Watermarks as Bibliographical Evidence," Neil Harris, Institut d'histoire du livre (website), 2017, http://ihl.enssib.fr/paper-and-watermarks-as-bibliographical-evidence. Methods dependent upon differences in paper density include tracing by hand over a light source; exposure to a photo-sensitive proofing paper such as DYLUX, largely discontinued; β-radiography; low energy χ-radiography, and reflected infrared (IR) and various multispectral imaging approaches (Harris 2017, pp.54-57). Methods dependent upon differences in paper topography include rubbing, ideally on the wire or mould side of the paper, with a soft pencil, largely discontinued; and Reflectance Transformation Imaging (RTI), K. Heumiller, S. Choi, J. Stenger, and C. Graham, "Post Processing of Reflectance Transform Imaging for Isolation of Surface Impressions," in *Archiving 2016 Final Program and Proceedings*, 2016, pp.15-20.
15. Digital photographs of eleven out of 87 full or partial watermarks found in the Codex Arundel were not available at the time of this writing.
16. Carmen Bambach, *Leonardo da Vinci Rediscovered*, New Haven and London, 2019, Vol.3, pp.273-274; also Vol.4, p.292, note 438: "The immediacy and specificity of description in Leonardo's references to Florence and to other sites in Tuscany suggest that the artist could well have begun the Codex Leicester while he was living in Florence at about the time that he was working on the Codex Arundel (British Library), which is dated on the first page 22 March 1508."
17. Bambach observed that the first 28 leaves of the Codex Arundel appear to have been originally a "unitary notebook or nestled loose sheets" and that Arundel 1v-14r once made up its "exterior cover or guard" (Bambach, Ref 16, Vol.4, p.327, n.50). Pedretti and Vecce observed similar affinities among the first thirty pages of the Codex (Pedretti and Vecce, Ref 1, Vol.4, p.53). Both references make note of a recurring cardinal's hat watermark; if Leonardo was using one batch of paper, this phenomenon would not be surprising.
18. William A. Sethares, Margaret Holben Ellis, and C. Richard Johnson, Jr. 2020, Ref 4.
19. The Codex Arundel is a quarto made from folded *rezzuta* format sheets; therefore approximately half of its folios contain a watermark.
20. The Codex Arundel is a quarto made from folded *rezzuta* format sheets; therefore approximately half of its folios contain a watermark.
21. Four cardinal's hat watermarks, three flower watermarks, two eagle watermarks, two scissors watermarks.
22. Bambach, Ref 16, Vol.4, Appendix I, p.1.
23. Also spelled *rezuta*. Brian Richardson, *Printing, Writers and Readers in Renaissance Italy*, Cambridge and New York, 1999, p.11; Harris, Ref 14, 2017, p.96.
24. The dimensions of the opened folios are approximately 300 x 400 mm (11 13/16" x 17 23/32"). Readers with an interest in 15th century paper are urged to investigate a tool for manuscript scholars and art historians that can be used to determine original paper formats: http://needhamcalculator.net

A note on the terms **bi-folio**, **folio**, and **sheet**:

Bi-folio: one sheet of paper in a codex, folded in half along the spine. Plural: **bi-folios.**

Folio: a leaf from a codex, typically one half of a bi-folio. Plural: **folios**. Folios are numbered according to the sequence that they appear in the codex. Each folio is typically still attached along the spine-fold to the other half of the bi-folio. For example, Arundel Folio 1 is attached along the spine-fold to Arundel Folio 14 (see Figure 5). Together, this folded piece of paper is referred to as Arundel Bi-folio 1v-14r, or simply Arundel 1v-14r. However, some folios in the Codex Arundel are not attached to any others. These folios are referred to as half sheets. For example, Arundel Half Sheet 15, or Arundel 15; Arundel Folio 15 is also accurate.

Sheet: Another word for the paper that makes up a bi-folio. In the Codex Leicester, each sheet is uncut, with four "deckle" edges remaining from the *rezzuta* format paper mould. Each sheet is still intact along the spine-fold. For example, folios 1 and 36 are attached; folios 2 and 35 are attached (see Figure 2). Therefore, the authors use the terms Leicester Sheet 1 and Leicester Sheet 2, instead of Leicester Bi-folio 1-36 and Leicester Bi-folio 2-35, although either is technically accurate. The uncomplicated collation of the Codex Leicester allows for this simplified terminology. See also, Bambach, Ref 16, Vol.4, Appendix III, pp.5-6.

25. Because a fleur-de-lis watermark was not present in the Codex Arundel for comparison and matching, Leicester Sheet 5 was not coded.
26. Victoria Blyth-Hill, "Conservation Treatment of a Leonardo da Vinci Manuscript," *Preprints of Papers Presented at the Tenth Annual Meeting*, Milwaukee, Wisconsin, 26-30 May, 1982, Washington, D.C.: American Institute for Conservation of Historic and Artistic Works, pp.113-123.
27. The recorded foliation can be seen in Plate 10.143, Bambach, Ref 16, Vol.3, p.268 and Blyth-Hill, Ref 26, p.113 as recorded by Gerolamo Calvi, *Il codice di Leonardo da Vinci (idraulica e cosmografia) della Biblioteca di Lord Leicester in Holkhan Hall, pubblicato sotto gli auspice del relae Istituto Lombardo di Scienze e Lettere, Milan, Premio Tomasoni*, Milan, 1909, and confirmed by Carlo Pedretti, *The Codex Hammer of Leonardo da Vinci*, translated into English and annotated by Carlo Pedretti, Florence, 1987, pp.182-83. See also Claire Farago, "The Codex Leicester," in *Leonardo da Vinci, Master Draftsman*, edited by Carmen C. Bambach, exh. cat., New York, NY, Metropolitan Museum of Art, 2003, pp.191-201.
28. Bambach, Ref 16, Vol.3, p.275.
29. Martin Kemp, "Compilation and Chaos," in Domenico Laurenza and Martin Kemp, *Leonardo da Vinci's Codex Leicester: A New Edition*, Oxford, 2019, Vol. II, pp.15-16.
30. Bambach, Ref 16, Vol.4, Appendix I, p.1.
31. The dimensions of the opened bi-folios are approximately 205 x 290 mm (8 3/32" x 11 14/32"); their edges have been trimmed. The quarto-sized Codex Arundel was made by folding in half a paper having a large *rezzuta* format paper, the same format as the sheets in the Codex Leicester. The folded paper is, in turn, folded in half again. In this arrangement, the sheet's watermark is located in the center fold line of the manuscript, where it is less likely to be completely obscured by writing.
32. See note 17.
33. Pedretti and Vecce, Ref 1, retain Arundel 18v (127) and Arundel 16r-17v (123) as among the first thirty bi-folios and half sheets of the Codex Arundel (115-134); Pedretti and Vecce, Ref 1, Vol.4, p.58.
34. Most recently, Alan Donnithorne, *Leonardo da Vinci: A Closer Look. Exploring the Beauty and Complexity of Leonardo's Drawings through a Study of his Materials and Methods*, London, 2019; Martin Clayton, *Leonardo da Vinci: A Life in Drawing*, London, 2018; Juliana Barone, ed., *Leonardo da Vinci: A Mind in Motion*, exh. cat., London, The British Library, 2019; Bambach, Ref 16; Michel Menu, ed., *Leonardo da Vinci's Technical Practice: Paintings, Drawings and Influence*, Paris, 2014; Benedetta Spadaccini, ed., *Leonardo da Vinci e il suo lascito: gli artisti e le tecniche*, exh. cat., Milan, Pinacoteca Ambrosiana, 2019; Musée du Louvre, *Léonard de Vinci: Dessins et manuscrits*, exh. cat., Paris, Musée du Louvre, 2003.
35. Carmen C. Bambach, "On the Role of Scientific Evidence in the Study of Leonardo's Drawings," in *Leonardo da Vinci's Technical Practice: Paintings, Drawings and Influence*, edited by Michel Manu, Paris, 2014, p.236.
36. As of this writing, two new sets of mouldmates have been identified in the Codex Arundel: Bull's head with flower above horns 161v-168r (112), 244v-247r (68), 279r-280v (145) and Hunter's horn 34v-35r (31), 122v-125r (32). (A looping overlay video of mouldmates Arundel 34v-35r and Arundel 122v-125r can be viewed at https://sethares.engr.wisc.edu/threads/hornWatermarkAnimation.gif).
37. See, for example, Elizabeth H. Bernick, "Drawing Connections – New Discoveries Regarding Cesare da Sesto's Sketchbook," *Master Drawings*, Vol.57, No.2, 2019, pp.147-196.
38. Line drawings are not suitable for computational coding or mouldmate matching. Three digital images are required: the recto and the verso of the sheet in visible light, and the sheet placed over a light source. Images of the entire sheet of paper in question, not just the area of the watermark, are optimal because they include more chain line intervals for measurement and comparison. All three digital images should be taken at a resolution sufficient for digital alignment, at least 300-600 DPI. The two surface images should be photographed under identical and even lighting. The transmitted light image should be lit from behind by bright and even illumination sufficient to reveal the internal features, but not so intense as to wash out their pale and subtle borders.
39. Twins, which are themselves mouldmates, can only be identified by their position within a sequence of other mouldmates.
40. The twin status of 208r-209v cannot be confirmed because its sequence relative to Leicester Sheets 14, 15, 17, and 18 cannot be determined.
41. Pedretti and Vecce, Ref 1, plate numbers in parentheses.
42. The eleven starred watermarks are not available digitally, but appear as small reproductions of radiographs in Pedretti and Vecce, Ref 1, "Apparati IV, V, Filigrane," pp.58-61. Radiographs were produced for all the watermarks reproduced in the Pedretti/Vecce publication and remain in the British Library. It is hoped to produce digital scans of the radiographs in the future.
43. Folio numbers have been transcribed according to Pedretti and Vecce, Ref 1.
44. Twins, which are themselves mouldmates, can only be identified by their position within a sequence of other mouldmates.
45. Twin status cannot be determined due to the disruption of the sequence.
46. The twin status of Sheet 6 is identified by its location within a sequence of mouldmates.
47. The twin status of Arundel 66r-67v cannot be confirmed because its sequence relative to Leicester Sheets 1, 2, and 4 is interrupted.
48. Sheet 3 is identified as a probable twin due to its location within a sequence of mouldmates.

IN MEMORIAM

STEFAN KAY MBE (1944-2021)

BAPH has sadly lost another remarkable member: Stefan George Kay MBE was one of that rare breed of people who, when talking to an old friend or a brand new acquaintance seemed to concentrate entirely on the person he was talking to. Of major importance in the paper making industry, he was devoted to his family and a man of many interests and enthusiasms.

At the start of the Second World War his father Stefan Kwiatkowski had escaped from Poland. (Figure 1) When the 1st Polish Parachute Brigade was formed in 1941, he joined up, becoming a First Lieutenant, and found himself in Peebles. There he met and married Barbara Taylor, the 17-year-old daughter of the local dentist. After the war he changed his name to Kay and became a paper mill accountant. Stefan was born in July 1944 and spent the early years of his life, along with his parents, in his grandparents' home, moving to Penicuik in 1951.

After education at St David's Dalkeith and Holy Cross Academy he became an engineering apprentice with Ferranti Ltd before completing a Mechanical Engineering degree at Heriot-Watt University after which he entered the paper industry in 1967 at Thames Board Ltd, where he became Technical Superintendent and then, after further training, Mill Chemist. He then returned to Scotland joining C H Dexter Ltd (now Ahlstrom-Munksjö Ltd) where he was Production Manager. In 1978 another move south took him to 10 years with St Regis Paper Company Ltd, as Mill Managing Director, first at New Taplow in Berkshire and then at Silverton Mills in Devon. He joined Inveresk PLC in 1988 as Managing Director; led a Management Buy-Out in 1990 and took the company to the London Stock Exchange in 1993. He retired from Inveresk PLC in 2001.

He was Chairman of several committees of the UK Paper Federation (now Confederation of Paper Industries): Energy (1982-1990), Environment (1992-4), and Forest Certification (1996-2000) Committees; and was President of the Paper Federation in 1994-6. He sat on the Confederation of European Paper Industries (CEPI) Board between 1994 and 1996; was Vice-Chairman then Chairman of the Environment Committee (1994-2000) and was again a Board member between 1998 and 2000. He chaired the CEPI Eco-Label Working Party between 1994 and 2002 and was a member of the Sustainability Working Party. He was awarded the Paper Industry Gold Medal in 1996.

During his time at Inveresk Stefan had become closely involved with his old university, Heriot-Watt, being a member of its Court for ten years from 1994, chairing the Staffing Strategy Committee and serving on the Finance Committee of Court. He was President of the Watt Club, thought to be the oldest alumni association in the UK, in 1994. He worked for Heriot-Watt University on a consultancy basis until 2003 and then as Director of Campus Services at their Galashiels Campus until 2009. In 2009 the University awarded him the Honorary Degree of Doctor of the University for services both to the University and to the Paper Industry.

Figure 1: *Stefan with his father in 1947 after the war.*